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On the convergence to the Nash bargaining solution for action-dependent bargaining protocols

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ABSTRACT

We consider a non-cooperative multilateral bargaining game and study an action-dependent bargaining protocol, that is, the probability with which a player becomes the proposer in a round of bargaining depends on the identity of the player who previously rejected. An important example is the frequently studied rejector-becomes-proposer protocol. We focus on subgame perfect equilibria in stationary strategies which are shown to exist and to be efficient. Equilibrium proposals do not depend on the probability to propose conditional on the rejection by another player. We consider the limit, as the bargaining friction vanishes. In case no player has a positive probability to propose conditional on his rejection, each player receives his utopia payoff conditional on being recognized. Otherwise, equilibrium proposals of all players converge to a weighted Nash bargaining solution, where the weights are determined by the probability to propose conditional on one's own rejection.

1. Introduction

This paper examines the convergence of equilibrium payoffs to the asymmetric Nash bargaining solution in a non-cooperative bargaining game. In contrast to the existing literature on this topic, we allow for the proposer selection process to be action-dependent, that is, influenced by the players' actions throughout the game.

The study of non-cooperative bargaining games has been strongly influenced by Rubinstein (1982). In his seminal paper, the unique division of a surplus among two impatient players is supported by subgame-perfect equilibrium. In unanimity bargaining games with more than two players, Herrero (1985) and Haller (1986) show that the uniqueness of subgame-perfect equilibrium payoffs is not preserved.[4] Therefore, the literature has focused attention on those subgame-perfect equilibria which are in stationary strategies.

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[4] One way to restore the uniqueness of subgame-perfect equilibrium is to deviate from unanimous agreement and consider instead a bargaining process where an agreement is reached in several steps and only a subset of the players bargain with each other at each step. Examples of such "partial agreements"

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The convergence of equilibrium payoffs in the limit as the cost of delay becomes small has been studied for a variety of different bargaining protocols, see Binmore et al. (1986), Hart and Mas-Colell (1996), Laruelle and Valenciano (2008), Miyakawa (2008), Britz et al. (2010), and Kultti and Vartiainen (2010). All those protocols have in common that they are action-independent: The actions taken by the players in the game have no effect on the identity of the next proposer. We argue that the focus on action-independent protocols alone is a serious limitation because action-dependent protocols are very common in other strands of the bargaining literature. One simple and intuitively appealing example is the protocol where the player who rejects the current proposal is automatically called to make the next proposal. This rejector-becomes-proposer protocol has been introduced in Selten (1981) and has been studied extensively in both the bargaining and the coalition formation literature, see for example Chatterjee et al. (1993), Bloch (1996), Ray and Vohra (1999), Imai and Salonen (2000), and Bloch and Diamantoudi (2011).

The protocol we study in this paper is more general than the rejector-becomes-proposer protocol. Following Kawamori (2008), we are interested in the case where the identity of the player who rejects a proposal may influence the probability by which a particular player becomes the next proposer. Since now the accept and reject decisions of the players influence the selection of the proposer, this leads to an action-dependent protocol. Such protocols are considerably more difficult to analyze than action-independent ones, and the literature has identified a number of cases where both types of protocol lead to surprisingly different results. For instance, Chatterjee et al. (1993) provide examples for non-existence of equilibria with immediate agreement in the context of an action-dependent protocol. On the contrary, it has been shown in Okada (1996) that delay cannot occur at equilibrium and in Okada (2011) that equilibria exist when the protocol is action-independent.[5]

Our analysis of stationary subgame perfect equilibria reveals that the set of equilibrium proposals only depends on the bargaining protocol through the probabilities of making counter-offers. A player's probability of making a counter-offer is defined as the probability for that player to become the proposer, given that the previous proposal has been rejected by *this* player.

We show that equilibrium proposals of all players converge to a weighted Nash bargaining solution, where the weights are proportional to the probabilities of making a counter-offer. One exception is the case when the probability of making a counter-offer is zero for all players. In this case the proposer in the initial round obtains his utopia payoff, that is his highest payoff in the set of feasible payoffs that satisfy all the individual rationality constraints. Equilibrium proposals of all players are independent of the continuation probability and do not converge to a common limit.

One implication of our analysis is that the probability of making a counter-offer is a crucial determinant of a player's bargaining power. The existing results on non-cooperative bargaining games are for action-independent protocols only and do not distinguish between the probability of making a proposal and the probability of making a proposal conditional on a rejection. Our paper identifies the latter probabilities as the source of bargaining power.

2. The bargaining game

We consider a bargaining game between finitely many players in the set $N = \{1, \dots, n\}$. Each player individually can only attain a disagreement payoff which we normalize to zero. However, the players can jointly achieve any payoff vector v in a set $V \subset \mathbb{R}^n$ if they unanimously agree on such a payoff vector. Each player is assumed to be an expected utility maximizer. The set V of feasible payoffs and the bargaining protocol are the main primitives of the model. We now introduce each in turn.

For vectors u and v in $\mathbb{R}^n$, we write $u \geqslant v$ if $u_i \geqslant v_i$ for all $i \in N$, $u > v$ if $u \geqslant v$ and $u \neq v$, and $u \gg v$ if $u_i > v_i$ for all $i \in N$. A point v of V is said to be *Pareto-efficient* if there is no point u in V such that $u > v$. A point v of V is said to be *weakly Pareto-efficient* if there is no point u in V such that $u \gg v$. We write V_+ to denote the set $V \cap \mathbb{R}^n_+$.

Our first assumption is as follows:

[A1] The set V is closed, convex, and comprehensive from below. There is a point $v \in V$ such that $v \gg 0$. The set V_+ is bounded. Each weakly Pareto-efficient point of V_+ is Pareto-efficient.

We denote the set of Pareto-efficient points of V by P and write P_+ for the set $P \cap \mathbb{R}^n_+$. A vector $\eta \in \mathbb{R}^n$ is called a *normal vector* to V at a point $v \in V$ if $(u - v)^\top \eta \leqslant 0$ for all $u \in V$. In addition, a normal vector to V at v is said to be a *unit normal vector* if $\|\eta\| = 1$.

[A2] There is a continuous function $\eta : P_+ \to \mathbb{R}^n$ such that $\eta(v)$ is a unit normal vector to V at the point v.

Assumption A2 implies that the boundary P_+ does not have kinks. Notice that in view of Assumption A1 we have $\eta_i(v) > 0$ for every $i \in N$ such that $v_i > 0$.

can be found in Chae and Yang (1994), Krishna and Serrano (1996), and Suh and Wen (2006). A similar approach has been applied to a coalition formation problem by Moldovanu and Winter (1995).

[5] Duggan (2011) presents a very general coalitional bargaining model where equilibrium existence is shown for action-independent protocols. The paper points out that a similar approach to establish equilibrium existence would not work when the protocol is action-dependent.

Bargaining takes place in discrete time $t = 0, 1, \ldots$. There are $n + 1$ probability distributions on the players denoted by $\pi^0, \pi^1, \ldots, \pi^n$, each of which belongs to the unit simplex Δ^n in $\mathbb{R}^n$.

In round $t = 0$, a particular player is chosen as the proposer according to the probability distribution $\pi^0 \in \Delta^n$. The proposer then makes a proposal $v \in V$. Player 1 responds to the proposal by either acceptance or rejection. Once a player $i = 1, \ldots, n-1$ has accepted the proposal, it is the turn of player $i + 1$ to accept or to reject.[6] Once player n has accepted the proposal, the game ends and the approved proposal is implemented.

As soon as some player $j \in N$ rejects a proposal in round t, the game ends with probability $1 - \delta > 0$ and payoffs to all players are zero. With the complementary probability δ, the game continues to round $t + 1$. The proposer in that round is then drawn from the probability distribution π^j. If the game continues perpetually without agreement, the payoff to every player is zero.

The rejector-becomes-proposer protocol follows from specifying $\pi^i_i = 1$ for all $i \in N$. A polar opposite of the rejector-becomes-proposer protocol, where a rejector proposes with probability zero in the next round, follows by setting $\pi^i_i = 0$ for all $i \in N$. In case $\pi^0, \pi^1, \ldots, \pi^n$ all coincide, we are back in the familiar case of an action-independent protocol with time-invariant recognition probabilities.

It is well-known that bargaining games with more than two players admit a wide multiplicity of subgame-perfect equilibria (SPE), see Herrero (1985) and Haller (1986). We will restrict attention to subgame-perfect equilibria in stationary strategies (SSPE). A *stationary strategy* for player i consists of a proposal $\theta^i \in V$ which i makes whenever it is his turn to propose and an acceptance set $A^i \subset V$ which consists of all the proposals which player i would be willing to accept if they were offered to him. We denote the *social acceptance set* by $A = \bigcap_{i \in N} A^i$ and write the profile of stationary strategies $(\theta^1, A^1, \ldots, \theta^n, A^n)$ more concisely as $(\Theta, \mathcal{A})$.

Consider some stationary strategy profile $(\Theta, \mathcal{A})$. The vector of *continuation payoffs* after i's rejection, denoted q^i, is the vector of payoffs in the subgame that follows the rejection of a proposal by some player $i \in N$. It is well-defined since $(\Theta, \mathcal{A})$ is stationary. Crucial for our analysis is the vector $r = (q^1_1, \ldots, q^n_n)$ of *reservation payoffs*. Under a protocol with time-invariant recognition probabilities, i.e. when $\pi^1 = \cdots = \pi^n$, one and the same vector of continuation payoffs would result no matter which player rejected the current proposal, i.e. $q^1 = \cdots = q^n$. Consequently, the reservation and continuation payoff vectors are equal to each other. If, however, we allow for an action-dependent protocol, the reservation payoff vector is in general not equal to any of the continuation payoff vectors. This disparity between the reservation and continuation payoffs is what makes the analysis of action-dependent protocols different from the analysis of action-independent protocols.

3. Convergence of SSPE payoffs

In order to study the convergence of SSPE payoffs when the breakdown probability tends to zero, we first state an existence result for SSPEs and a characterization of the associated proposals.

Theorem 3.1. *An SSPE exists. In every SSPE, agreement is reached immediately, and the proposals are such that for every $i \in N$,*

$$\theta^i \in P_+, \tag{3.1}$$

$$\theta^i_j = \alpha_j \theta^j_j, \quad j \in N \setminus \{i\}, \tag{3.2}$$

where $\alpha_j = \delta\pi^j_j/(1 - \delta + \delta\pi^j_j)$. Conversely, for every profile of proposals $(\theta^1, \ldots, \theta^n)$ satisfying (3.1)–(3.2), there is an SSPE with this profile of proposals.

Proof. For a detailed derivation of the existence and characterization theorem, we refer to the working paper by Britz et al. (2013). For existence of an SSPE, one has to show that the system consisting of Eqs. (3.1)–(3.2) has a solution. The argument can be easily adapted from Banks and Duggan (2000). The proof that for every profile of proposals $(\theta^1, \ldots, \theta^n)$ satisfying (3.1)–(3.2) there is an SSPE with this profile of proposals is standard but tedious. Here, we restrict ourselves to the proof of the immediate acceptance property and the derivation of (3.1)–(3.2), where we focus on the parts of the proof which are different from the action-independent case. In what follows we fix an SSPE $(\Theta, \mathcal{A})$.

Step 1. *For each $v \in V$: If $v \in V$ is such that $v \gg r$, then $v \in A$. If $v \in A$, then $v \geqslant r$.*

The proof of this step is easy and therefore omitted.

Step 2. *The social acceptance set A is non-empty and $r \geqslant 0$. There exists $v \in V$ such that $v \gg r$.*

[6] Throughout the paper, for the sake of simplicity, we assume that players respond to a proposal in the fixed order $1, \ldots, n$. All results would carry over to the case with arbitrary voting orders.

Suppose there is no $v \in V$ such that $v \gg r$. In view of Assumption A1, there is no $v \in V$ such that $v > r$. It now follows from Step 1 that $A \subset \{r\}$. First suppose that $A = \emptyset$. In this case equilibrium strategies lead to payoffs of zero for all players, so $r = 0$. Under Assumption A1 there is a vector $v \in V$ with $v \gg 0$, a contradiction to our supposition. Hence $A = \{r\}$.

Then, after a rejection, only two outcomes are possible: Either agreement on r is reached at some future time or zero payoffs result. The vector of players' continuation payoffs after a rejection is therefore a convex combination of 0 and r, where the former has a weight of at least $1 - \delta$. But this implies $r_i \leqslant \delta r_i$ for all $i \in N$. Since $\delta < 1$, we conclude that $r = 0$. As before, this leads to a contradiction.

We conclude that there is $v \in V$ such that $v \gg r$. Step 1 implies that $v \in A$. Since each player can choose to reject all proposals, $r \geqslant 0$.

Step 3. *For every $i \in N$, $\theta^i \in A$.*

Let u_i be the SSPE utility to player i at a history where it is player i's turn to make a proposal. It holds that $u_i = \theta^i_i$ if $\theta^i \in A$ and $u_i = q^j_i$ if $\theta^i \notin A$, where j is the least element of N such that $\theta^i \notin A^j$.

By making a proposal $v \in A$, player i guarantees himself a payoff of v_i. It follows that $u_i \geqslant v_i$ for every $v \in A$. In particular $u_i > 0$ since by Step 2 there is a vector $v \in A$ such that $v_i > 0$.

Let $U = \{0\} \cup (A \cap \{\theta^1, \dots, \theta^n\})$. This is the set of all possible outcomes of the game if play follows the strategy $(\Theta, \mathcal{A})$. We know from the preceding paragraph that $u_i \geqslant v_i$ for all $v \in U$ and that $u_i > 0$. Take any $k \in N$. The vector q^k of continuation payoffs is a convex combination of the vectors in U, with 0 having a weight of at least $1 - \delta$. It follows that $u_i > q^k_i$. Since this holds for each $k \in N$, we must have $u_i = \theta^i_i$ and $\theta^i \in A$.

Step 4. *For every $i \in N$, $\theta^i \in P_+$ and $\theta^i_j = r_j$ for every $j \in N \backslash \{i\}$.*

The proof follows a standard argument and is therefore omitted.

Step 5. *For every $i \in N$, for every $j \in N \setminus \{i\}$, $\theta^i_j = \alpha_j \theta^j_j$.*

Since each proposal belongs to the social acceptance set, the reservation payoffs can be computed as follows:

$$r_j = \delta \sum_{k=1}^{n} \pi^j_k \theta^k_j = \delta \pi^j_j \theta^j_j + \delta\big(1 - \pi^j_j\big) r_j.$$

Solving for r_j, we see that $r_j = \alpha_j \theta^j_j$. Combining this with Step 4 we obtain the result. □

In the existing literature on n-player bargaining games, it has been shown for a variety of action-independent protocols that the SSPE proposals of all players converge to a common limit proposal. We are interested in the question whether this result carries over to an action-dependent protocol. Indeed, it turns out that the existence of a limit equilibrium proposal is preserved under an action-dependent protocol, except in the degenerate case where the probability of proposing after one's own rejection is zero for all players.

For $m \in \mathbb{N}$, let $(\delta_m)_{m \in \mathbb{N}}$ be a sequence of continuation probabilities converging to 1 and let Θ_m be SSPE equilibrium proposals of the game with continuation probability δ_m. Equilibrium proposals are said to converge to each other if for all $i, j \in N$ it holds that $\|\theta^i_m - \theta^j_m\|_\infty \to 0$.

Proposition 3.2.

1. *If $\pi^j_j = 0$ for all $j \in N$, then equilibrium proposals do not converge to each other.*
2. *If there is $j \in N$ such that $\pi^j_j > 0$, then equilibrium proposals do converge to each other.*

Proposition 3.2 can be derived from the SSPE characterization. We define $\hat{v}$ as the vector of *utopia payoffs*, where the utopia payoff of a player $i \in N$ is the highest payoff in V for player i that satisfies all the individual rationality constraints, that is $\hat{v}_i = \max\{v_i \in \mathbb{R} \mid v \in V_+\}$. Consider the case where $\pi^j_j = 0$ for all $j \in N$. In that case, we have $\alpha_j = 0$ and hence $r_j = 0$ for all $j \in N$. It follows that $\theta^i_j = 0$ for all $j \in N$ and $i \in N \setminus \{j\}$. Due to the Pareto-efficiency of all proposals, it must be true that $\theta^j_j = \hat{v}_j$ for all $j \in N$. This argument does not depend on the value of δ. SSPE proposals do not converge to each other. Now consider the case where there exists $j \in N$ such that $\pi^j_j > 0$. Then, we see that α_j converges to one in the limit as δ goes to one. Thus in the limit we have that all players $j \in N$ such that $\pi^j_j > 0$ are offered the same payoff by every proposer. All other players are offered a zero payoff when they are responding, and the Pareto-efficiency of all proposals implies that they also receive zero when they are proposers.

Every accumulation point of SSPE proposals when δ tends to one is called a *limit equilibrium proposal*. We proceed by showing that if there is $j \in N$ such that $\pi^j_j > 0$, then the limit equilibrium proposal is unique and equal to the asymmetric Nash bargaining solution where player i has weight π^i_i. Given a vector $\lambda \in \mathbb{R}^n_+ \setminus \{0\}$, we define the λ-Nash product $\rho_\lambda : \mathbb{R}^n_+ \to \mathbb{R}$ by

$$\rho_\lambda(v) = \prod_{i \in N} v_i^{\lambda_i}, \quad v \in \mathbb{R}^n.$$

Definition 3.3. Given a vector $\lambda \in \mathbb{R}^n_+ \setminus \{0\}$, the maximizer of the function ρ_λ on V_+ is called the λ-Nash bargaining solution.

Under our assumptions, the maximizer of the function ρ_λ on V_+ is indeed unique. It is a Pareto-efficient point of V which is uniquely characterized by the following conditions: for each i and j in N,

$$\text{if } \lambda_i = 0, \quad \text{then } v_i = 0, \tag{3.3}$$

$$\text{if } \lambda_i, \lambda_j > 0, \quad \text{then } \frac{v_i \eta_i(v)}{\lambda_i} = \frac{v_j \eta_j(v)}{\lambda_j}. \tag{3.4}$$

Theorem 3.4.

1. *If $\pi^j_j = 0$ for all $j \in N$, then each player's expected payoff is $\pi^0_j \hat{v}_j$.*
2. *If there is $j \in N$ such that $\pi^j_j > 0$, then the limit equilibrium proposal is unique and is equal to the $(\pi^1_1, \ldots, \pi^n_n)$-Nash bargaining solution.*

Proof. The first part follows from Proposition 3.2, we show the second part. We verify that each limit equilibrium proposal satisfies the conditions (3.3)–(3.4) with $\lambda_i = \pi^i_i$. Let

$$\tilde{N} = \{ i \in N \mid \pi^i_i > 0 \}.$$

Let $(\theta^1, \ldots, \theta^n)$ be SSPE proposals in a game with continuation probability δ. By the definition of the normal vector it holds for any two players i and j that

$$(\theta^j - \theta^i)^\top \eta(\theta^i) \leqslant 0.$$

Notice that the proposals θ^i and θ^j can only differ in components i and j. Solving for the inner product, we can therefore rewrite the previous inequality as

$$(\theta^j_j - \theta^i_j)\eta_j(\theta^i) + (\theta^j_i - \theta^i_i)\eta_i(\theta^i) \leqslant 0.$$

Substituting for θ^i_j and θ^j_j from Eq. (3.2) and dividing by $1 - \delta$ yields

$$\frac{\theta^j_j \eta_j(\theta^i)}{1 - \delta + \delta \pi^j_j} \leqslant \frac{\theta^i_i \eta_i(\theta^i)}{1 - \delta + \delta \pi^i_i}.$$

Let $\bar{\theta}$ be a limit equilibrium proposal. Taking the limit of the latter inequality along a sequence of equilibrium proposals converging to $\bar{\theta}$, we obtain for all $i, j \in \tilde{N}$,

$$\frac{\bar{\theta}_j \eta_j(\bar{\theta})}{\pi^j_j} \leqslant \frac{\bar{\theta}_i \eta_i(\bar{\theta})}{\pi^i_i}.$$

Interchanging the roles of the players i and j, we obtain the equality

$$\frac{\bar{\theta}_j \eta_j(\bar{\theta})}{\pi^j_j} = \frac{\bar{\theta}_i \eta_i(\bar{\theta})}{\pi^i_i}, \quad i, j \in \tilde{N}. \tag{3.5}$$

This shows that $\bar{\theta}$ satisfies (3.4). The fact that $\bar{\theta}$ satisfies (3.3) follows at once from the fact that a player $k \in N$ with $\pi^k_k = 0$ receives a zero payoff irrespective of δ so that $\bar{\theta}_k = 0$. □

It is remarkable that the probability to become the proposer after another player's rejection does not matter for the limit equilibrium payoffs. This is in contrast to the analogous results for action-independent protocols. For instance, Britz et al. (2010) study a multilateral bargaining game in which the proposer is selected according to a Markov process. That is, the probability distribution from which the proposer at round $t+1$ is drawn depends on the identity of the proposer in round t. In that case, the limit equilibrium corresponds to an asymmetric Nash bargaining solution where the vector of bargaining weights is given by the stationary distribution of the Markov process. This Markov process is represented by an $n \times n$-matrix, and all entries of this matrix influence the limit equilibrium payoffs.

4. Conclusion

We have considered multilateral bargaining games with action-dependent protocols. The identity of the player who rejects the current proposal determines the probability distribution from which the next proposer is drawn. The probability with which a player proposes after his own rejection is crucial for the equilibrium prediction. In particular, if all players have the same positive probability of proposing after their own rejection, we find convergence to the (symmetric) Nash bargaining solution.

One rather surprising outcome of our analysis is that the probability to become proposer conditional on the rejection of another player does not affect equilibrium payoffs at all. This also sheds new light on the findings in Miyakawa (2008) and Laruelle and Valenciano (2008) concerning the protocol with time-invariant recognition probabilities, which is a special case of our model. Under time-invariant recognition probabilities, it is impossible to discern the effect of the probability to become proposer after one's own rejection as opposed to the probability to propose after another player's rejection. Our more general setup makes the importance of this distinction apparent.

We show that the convergence to the asymmetric Nash bargaining solution breaks down in the degenerate case where each player has zero probability of proposing after his own rejection. In that case, the equilibrium payoffs are determined by the utopia point and the recognition probabilities in the initial round.

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