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Pareto-improving price regulation when the asset market is incomplete: An example

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In a robust example of an economy with an incomplete asset market, price regulation, that operates anonymously, on market variables, can be Pareto-improving.

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1 Introduction

When all contracts for the transfer of revenue across realizations of uncertainty are traded in competitive markets to which all individuals have unrestricted access, the asset market is complete; otherwise, the asset market is incomplete.

An incomplete asset market, typically, fails to make optimal use of the limited menu of assets at its disposal. Geanakoplos and Polemarchakis (1986) showed that, generically, there exist reallocations of portfolios that yield Pareto improvements in welfare after prices in spot commodity markets adjust to attain equilibrium.

The failure of constrained optimality casts doubt on non-intervention with competitive markets.

An alternative to the reallocation of asset portfolios is the direct regulation of prices in spot commodity markets. An extension of the fix-price equilibrium of Drèze (1975) provides the required notion of equilibrium that allows for trading at non-competitive prices.

An intervention in spot market prices is not an intervention in individual choice variables, but in market variables; and it satisfies the requirement of anonymity. Nevertheless, concerns about the informational requirements for the determination, even computation, of improving interventions remain. These requirements involve the marginal utilities of

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income and excess demands for commodities across states. Carvajal and Polemarchakis (2008), Carvajal and Riascos (2005), Geanakoplos and Polemarchakis (1990) and Kübler, Chiappori, Ekeland and Polemarchakis (2002) are only first steps towards the identifications of these magnitudes from market data.

In Herings and Polemarchakis (2005), we showed that, if fix-price equilibria behave sufficiently regularly near Walrasian equilibria, Pareto-improving price regulation is generically possible.

Here, we develop an explicit and robust example that yields globally unique fix-price equilibria. When fix-price equilibria are globally unique, the fix-price equilibrium manifold behaves well enough, even at competitive equilibrium prices, to allow for the application of differential calculus and Pareto-improving price regulation is possible.

In our example, the cardinal utility indices are quasi-linear, and this requires restrictions on parameter values to guarantee differentiable comparative statics; nevertheless, endowments vary in an open set.

Antecedents of this result are the argument in Polemarchakis (1979), where fixed wages that need not match shocks in productivity may yield higher expected utility in spite of the loss of output in an economy of overlapping generations; and the argument in Drèze and Gollier (1993), that employs the capital asset pricing model to determine optimal schedules of wages that differ from the marginal productivity of labor. Kalmus (1997) gave a heuristic example of Pareto improving price regulation.

Drèze and Herings (2008) explain price stickiness as a consequence of kinked perceived demand curves. This paper motivates price stickiness as a Pareto-improving policy response in the presence of incomplete markets.

2 The example

The economy is that of the standard two-period general equilibrium model with numéraire assets and an incomplete asset market. Assets exchange before and commodities after the state of the world realizes.

There are two individuals, $i = 1, 2$ (or i, i'), three states of the world, $s = 1, 2, 3$, two commodities, 1, 2 and two assets, 1, 2.

The utility function of individual i has an additively separable representation, $u^i = \sum_{s \in \mathcal{S}} \pi_s u^i_s$, with state-dependent cardinal utility

$$u^i_s(x_s) = \alpha^i_s \ln x_{1,s} + \beta^i_s x_{2,s}, \quad \alpha^i_s > 0, \beta^i_s > 0,$$

and a strictly positive probability measure (π_1, π_2, π_3) over the states of the world; his endowment, $e^i = (e^i_{1,1}, e^i_{2,1}, e^i_{1,2}, e^i_{2,2}, e^i_{1,3}, e^i_{2,3})$, is strictly positive.

Notice that, at every state of the world, the cardinal utility u^i_s is quasi-linear: it is linear in $x_{2,s}$. This feature enables us to derive equilibria explicitly, even in an incomplete markets framework, where explicit solutions are otherwise hard to obtain. But, it requires parameter restrictions and often ad hoc arguments.

Prices of commodities at a state of the world are $p_s = (p_{1,s}, 1) \gg 0$: at every state of the world, commodity 2 is numéraire, and its price is $p_{2,s} = 1$; across states of the world,

$p = (\ldots, p_s, \ldots) \gg 0$. Prices of assets are $q = (q_1, 1)$; asset 2 is numéraire, and its price is $q_2 = 1$.

The payoffs of assets, denominated in the numéraire commodity at every state of the world, are

$$R = (r_1, r_2) = \begin{pmatrix} \vdots \\ \\ R_s \\ \\ \vdots \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ \\ 0 & 1 \\ \\ 0 & 1 \end{pmatrix};$$

that is, asset 1 delivers one unit of the numéraire commodity 2 in state 1, and nothing in other states; while asset 2 delivers one unit of commodity 2 in states 2 and 3 and nothing in state 1. The asset structure allows for the following interpretation. Consumption at state of the world 1 corresponds to first period consumption. Asset 2 is traded against asset 1 or against first period consumption. Asset 2 is an indexed bond with state-independent payoffs.

Arbitrary commodity and asset prices are typically incompatible with a competitive equilibrium. In commodities and assets other than the numéraire, rationing on net trades, uniform across individuals, serves to attain market clearing. Rationing in the supply (demand) of commodities other than the numéraire is $\underline{z} = (\ldots, \underline{z}_{1,s}, \ldots) \leq 0$ ($\overline{z} \geq 0$). Rationing in the supply (demand) of asset 1 is $\underline{y} \leq 0$ ($\overline{y} \geq 0$).

At prices and rationing $(p, q, \underline{z}, \overline{z}, \underline{y}, \overline{y})$, the budget set of an individual is

$$\beta^i(p, q, \underline{z}, \overline{z}, \underline{y}, \overline{y}) = \left\{ (x, y) : \begin{array}{l} qy \leq 0, \\ p_s\big(x_s - e_s^i\big) \leq R_s y, \\ \underline{z}_{1,s} \leq x_{1,s} - e_{1,s}^i \leq \overline{z}_{1,s}, \\ \underline{y}_1 \leq y \leq \overline{y}_1, \end{array} \right\};$$

his optimization problem is to choose a utility-maximizing consumption bundle and asset portfolio in his budget set. The set of utility-maximizing consumption bundles and asset portfolios is denoted $d^i(p, q, \underline{z}, \overline{z}, \underline{y}, \overline{y})$.

An individual is effectively rationed in his supply (demand) for a commodity or an asset if he could increase his utility when the rationing constraint in the supply (demand) of that commodity or asset is removed. There is effective supply (demand) rationing in the market for a commodity or an asset if at least one individual is effectively rationed in his supply (demand) for this commodity or asset. At a competitive equilibrium, there is neither effective supply rationing nor effective demand rationing in the market for any commodity or asset. In this sense, a competitive equilibrium is a special case of a fix-price equilibrium.

Definition 1 (Fix-price equilibrium)

A fix-price equilibrium at prices (p, q) *is a pair* $((x^*, y^*), (\underline{z}^*, \overline{z}^*, \underline{y}^*, \overline{y}^*))$, *such that*

(1) *for every individual,* $(x^{i*}, y^{i*}) \in d^i(p, q, \underline{z}^*, \overline{z}^*, \underline{y}^*, \overline{y}^*)$,

(2) $\sum_{i=1}^{2} x^{i*} = \sum_{i=1}^{2} e^i$ *and* $\sum_{i=1}^{2} y^{i*} = 0$,

(3) *if, for* i', $x^{i'*}_{1,s} - e^{i'}_{1,s} = \underline{z}^*_{1,s}$, *then, for all* i, $x^{i*}_{1,s} - e^{i}_{1,s} < \overline{z}^*_{1,s}$, *while, if, for* i', $x^{i'*}_{1,s} - e^{i'}_{1,s} = \overline{z}^*_{1,s}$, *then, for all* i, $x^{i*}_{1,s} - e^{i}_{1,s} > \underline{z}^*_{1,s}$,

(4) *if, for* i', $y^{i'*}_1 = \underline{y}^*_1$, *then, for all* i, $y^{i*}_1 < \overline{y}^*_1$, *while, if for* i', $y^{i'*}_1 = \overline{y}^*_1$, *then, for all* i, $y^{i*}_1 > \underline{y}^*_1$.

The linearity of the utility functions in $x^i_{2,s}$ leads to extreme choices of asset portfolios and to non-differentiabilities unless the ratio of prices of the assets equals the corresponding ratio of the inner product of the state-dependent marginal utility of income with the asset payoff vector; and this, evidently, for every individual. To guarantee this equality, we choose the parameters in the utility functions of individuals such that

$$\pi = \frac{\pi_1 \beta^1_1}{\pi_2 \beta^1_2 + \pi_3 \beta^1_3} = \frac{\pi_1 \beta^2_1}{\pi_2 \beta^2_2 + \pi_3 \beta^2_3}.$$

At the price $q = 1/\pi$ for the indexed bond, individuals are indifferent with respect to the choice of their asset portfolio.

For the application of differential calculus, it is essential that competitive equilibria be interior. In order to eliminate equilibria at the boundaries of the consumption sets of individuals, we choose their endowments such that

$$\max \left\{ -e^1_{2,s} + \frac{\gamma^1_s e^2_{1,s} - \gamma^2_s e^1_{1,s}}{e^1_{1,s} + e^2_{1,s}} : s = 2, 3, -\pi e^2_{2,1} + \pi \frac{\gamma^2_1 e^1_{1,1} - \gamma^1_1 e^2_{1,1}}{e^1_{1,1} + e^2_{1,1}} \right\}$$

$$\leq \min \left\{ \pi e^1_{2,1} + \pi \frac{\gamma^2_1 e^1_{1,1} - \gamma^1_1 e^2_{1,1}}{e^1_{1,1} + e^2_{1,1}}, e^2_{2,s} + \frac{\gamma^1_s e^2_{1,s} - \gamma^2_s e^1_{1,s}}{e^1_{1,s} + e^2_{1,s}} : s = 2, 3 \right\},$$

where

$$\gamma^i_s = \alpha^i_s / \beta^i_s.$$

For the parameters chosen so far,[1] we will show next that fix-price equilibria are locally unique in the neighborhood of the competitive equilibrium, and that, generically in initial endowments, it is possible to achieve Pareto improvements by appropriately regulating the prices of commodities.

[1] A possible choice of parameters is, for instance,

$$\pi_1 = 1, \pi_2 = \pi_3 = 1/2,$$
$$\alpha^1_1 = \beta^1_1 = 1, \alpha^1_2 = \beta^1_2 = 4/3, \alpha^1_3 = \beta^1_3 = 2/3,$$
$$\alpha^2_1 = \beta^2_1 = 1, \alpha^2_2 = \beta^2_2 = 2/3, \alpha^2_3 = \beta^2_3 = 4/3,$$
$$e^1_1 = (1, 1), e^1_2 = (1, 1), e^1_3 = (2, 1),$$
$$e^2_1 = (1, 1), e^2_2 = (2, 1), e^2_3 = (1, 1).$$

Demand for commodity 1 of individual i in state s equals γ_s^i/p_s. The specification of our utility functions ensures that this demand does not depend on prices in other states. It follows easily that competitive equilibrium prices are given by

$$p_s^* = \frac{\gamma_s^1 + \gamma_s^2}{e_{1,s}^1 + e_{1,s}^2},\ s = 1, 2, 3, \quad \text{and} \quad q^* = \frac{1}{\pi}.$$

We fully characterize fix-price equilibria where the price of the indexed bond is chosen to equal $q = 1/\pi$. This avoids rationing in the asset market. Fix-price equilibrium exists for all prices of commodities, p. Given a state s, we label individuals i and i' such that $\gamma_s^i/e_{1,s}^i \leq \gamma_s^{i'}/e_{1,s}^{i'}$. As a percentage of initial endowment of commodity 1, agent i' has higher demand for commodity 1 than agent i. There are four different cases:

(i) $0 < p_s \leq \gamma_s^i/e_{1,s}^i$,
(ii) $\gamma_s^i/e_{1,s}^i \leq p_s \leq (\gamma_s^i + \gamma_s^{i'})/(e_{1,s}^i + e_{1,s}^{i'})$,
(iii) $(\gamma_s^i + \gamma_s^{i'})/(e_{1,s}^i + e_{1_s}^{i'}) \leq p_s \leq \gamma_s^{i'}/e_{1,s}^{i'}$ and
(iv) $\gamma_s^{i'}/e_{1,s}^{i'} \leq p_s$.

In case (i), both individuals have excess demand for commodity 1. The fix-price equilibrium is a trivial one, where both individuals are fully rationed on their demand for commodity 1 and consume their initial endowments of this commodity. Consumption of commodity 2 is equal to the initial endowments plus the payoffs of the asset portfolio chosen. In case (iv), both individuals have excess supply of commodity 1. The fix-price equilibrium is again a trivial one, where both individuals are fully rationed on their supply of commodity 1, and consume their initial endowments of commodity 1. Consumption of commodity 2 equals the initial endowments plus the payoffs of the asset portfolio. The most interesting cases are cases (ii) and (iii). Competitive prices belong to the intersection of cases (ii) and (iii).

In case (ii), with $\gamma_s^i/e_{1,s}^i \leq p_s \leq (\gamma_s^i + \gamma_s^{i'})/(e_{1,s}^i + e_{1,s}^{i'})$, there is aggregate excess demand for commodity 1, but individual i is a net supplier of the commodity, and trade takes place, with individual i' rationed on his demand for the commodity. Rationing on demand is determined by the net supply of individual i, so equilibria obtain for

$$\begin{aligned}
\overline{z}_{1,s}^* &= e_{1,s}^i - \gamma_s^i/p_s,\\
x_{1,s}^{i*} &= \gamma_s^i/p_s,\\
x_{1,s}^{i'*} &= e_{1,s}^{i'} + e_{1,s}^i - \gamma_s^i/p_s,\\
y^{i'*} &= -y^{i*}.
\end{aligned}$$

Straightforward substitutions yield that at $s = 1$,

$$\begin{aligned}
x_{2,1}^{i*} &= p_1 e_{1,1}^i + e_{2,1}^i - \gamma_1^i - (1/\pi)y^{i*},\\
x_{2,1}^{i'*} &= e_{2,1}^{i'} - p_1 e_{1,1}^i + \gamma_1^i + (1/\pi)y^{i*},
\end{aligned}$$

where non-negativity constraints on consumption imply that y^{i*} is restricted by

$$y^{i*} \leq \pi\left(p_1 e^i_{1,1} + e^i_{2,1} - \gamma^i_1\right),$$
$$y^{i*} \geq -\pi\left(e^{i'}_{2,1} - p_1 e^i_{1,1} + \gamma^i_1\right).$$

At $s = 2$ or $s = 3$,

$$x^{i*}_{2,s} = p_s e^i_{1,s} + e^i_{2,s} - \gamma^i_s + y^{i*},$$
$$x^{i'*}_{2,s} = e^{i'}_{2,s} - p_s e^i_{1,s} + \gamma^i_s - y^{i*},$$

where non-negativity constraints on consumption imply that y^{i*} is restricted by

$$y^{i*} \geq -p_s e^i_{1,s} - e^i_{2,s} + \gamma^i_s,$$
$$y^{i*} \leq e^{i'}_{2,s} - p_s e^i_{1,s} + \gamma^i_s.$$

The restrictions we have imposed on initial endowments guarantee that a choice of y^{i*} satisfying all the specified inequalities is feasible. The remaining parameters of the rationing scheme are set so as not to be binding.

In case (iii), with $(\gamma^i_s + \gamma^{i'}_s)/(e^i_{1,s} + e^{i'}_{1,s}) \leq p_s \leq \gamma^{i'}_s/e^{i'}_{1,s}$, there is aggregate excess supply of commodity 1, and individual i is a net supplier of the commodity, rationed by the net demand of individual i'. Equilibria obtain for

$$\underline{z}^*_{1,s} = e^{i'}_{1,s} - \gamma^{i'}_s/p_s,$$
$$x^{i*}_{1,s} = e^i_{1,s} + e^{i'}_{1,s} - \gamma^{i'}_s/p_s,$$
$$x^{i'*}_{1,s} = \gamma^{i'}_s/p_s,$$
$$y^{i'*} = -y^{i*}.$$

Straightforward substitutions yield that at $s = 1$,

$$x^{i*}_{2,1} = e^i_{2,1} - p_1 e^{i'}_{1,1} + \gamma^{i'}_1 - (1/\pi)y^{i*},$$
$$x^{i'*}_{2,1} = p_1 e^{i'}_{1,1} + e^{i'}_{2,1} - \gamma^{i'}_1 + (1/\pi)y^{i*},$$

where non-negativity constraints on consumption imply that y^{*i} is restricted by

$$y^{i*} \leq \pi\left(e^i_{2,1} - p_1 e^{i'}_{1,1} + \gamma^{i'}_1\right),$$
$$y^{i*} \geq -\pi\left(p_1 e^{i'}_{1,1} + e^{i'}_{2,1} - \gamma^{i'}_1\right).$$

At $s = 2$ or $s = 3$,

$$x^{i*}_{2,s} = e^i_{2,s} - p_s e^{i'}_{1,s} + \gamma^{i'}_s + y^{i*},$$
$$x^{i'*}_{2,s} = p_s e^{i'}_{1,s} + e^{i'}_{2,s} - \gamma^{i'}_s - y^{i*},$$

where non-negativity constraints on consumption imply that y^{*i} is restricted by

$$y^{i*} \geq -e^{i}_{2,s} + p_s e^{i'}_{1,s} - \gamma^{i'}_s,$$
$$y^{i*} \leq p_s e^{i'}_{1,s} + e^{i'}_{2,s} - \gamma^{i'}_s.$$

The restrictions we have imposed on initial endowments guarantee that a choice of y^{i*} satisfying all the specified inequalities is feasible. The remaining parameters of the rationing scheme are set so as not to be binding.

The utility attained by each individual at a fix-price equilibrium is unambiguously determined by the prices of commodities. It is in this sense that the fix-price equilibria are globally unique, so certainly locally unique in the neighborhood of a competitive equilibrium. Since we are interested in price regulations in the neighborhood of competitive equilibrium prices, we may restrict ourselves to cases (ii) and (iii). At prices p, the indirect utility of individual i at the fix-price equilibrium is $v^i(p) = \sum_{s\in\mathcal{S}} \pi_s v^i_s(p_s)$, where

$$v^i_s(p_s) = \alpha^i_s \ln\left(\frac{\gamma^i_s}{p_s}\right) + \beta^i_s\big(p_s e^i_{1,s} + e^i_{2,s} - \gamma^i_s\big),$$

$$v^{i'}_s(p_s) = \alpha^{i'}_s \ln\left(e^{i'}_{1,s} + e^{i}_{1,s} - \frac{\gamma^i_s}{p_s}\right) + \beta^{i'}_s\left(e^{i'}_{2,s} - p_s e^i_{1,s} + \gamma^i_s\right)$$

if $\gamma^i_s/e^i_{1,s} \leq p_s \leq (\gamma^i_s + \gamma^{i'}_s)/(e^i_{1,s} + e^{i'}_{1,s})$, case (ii), while

$$v^i_s(p_s) = \alpha^i_s \ln\left(e^i_{1,s} + e^{i'}_{1,s} - \frac{\gamma^{i'}_s}{p_s}\right) + \beta^i_s\big(e^i_{2,s} - p_s e^{i'}_{1,s} + \gamma^{i'}_s\big),$$

$$v^{i'}_s(p_s) = \alpha^{i'}_s \ln\left(\frac{\gamma^{i'}_s}{p_s}\right) + \beta^{i'}_s\left(p_s e^{i'}_{1,s} + e^{i'}_{2,s} - \gamma^{i'}_s\right)$$

if $(\gamma^i_s + \gamma^{i'}_s)/(e^i_{1,s} + e^{i'}_{1,s}) \leq p_s \leq \gamma^{i'}_s/e^{i'}_{1,s}$, case (iii).

Substitution of the competitive equilibrium prices in either case (ii) or case (iii) gives the utility levels at the competitive equilibrium. The indirect utility function is differentiable at competitive prices, and the derivative is given by

$$\partial_{p_s} v^i(p^*) \ = \pi_s \beta^i_s \frac{\gamma^{i'}_s e^i_{1,s} - \gamma^i_s e^{i'}_{1,s}}{\gamma^i_s + \gamma^{i'}_s} \ = -\pi_s \beta^i_s\big(x^{i*}_s - e^i_s\big),$$

$$\partial_{p_s} v^{i'}(p^*) = \pi_s \beta^{i'}_s \frac{\gamma^i_s e^{i'}_{1,s} - \gamma^{i'}_s e^i_{1,s}}{\gamma^{i'}_s + \gamma^i_s} = -\pi_s \beta^{i'}_s\left(x^{i'*}_s - e^{i'}_s\right).$$

The important fact to notice is that at competitive equilibrium prices, the derivative of the indirect utility function of individual i with respect to the price of a commodity l in a state s equals the product of the marginal utility of income of individual i in state s multiplied by the excess supply of individual i of commodity l in state s. It is shown in Herings and Polemarchakis (2005) that this is a rather general feature. For instance, local uniqueness of fix-price equilibria in the neighborhood of a competitive equilibrium, together with standard differentiability assumptions, suffices. As a consequence, it is only the first-order effect of a price regulation that matters, whereas the negative effects on utility caused by rationing constraints are only second-order effects.

For $v_s = \pi_s(\gamma_s^2 e_{1,s}^1 - \gamma_s^1 e_{1,s}^2)/(\gamma_s^1 + \gamma_s^2)$,

$$V = \begin{pmatrix} \partial v^1(p^*) \\ \partial v^2(p^*) \end{pmatrix} = \begin{pmatrix} \beta_1^1 v_1 & \beta_2^1 v_2 & \beta_3^1 v_3 \\ -\beta_1^2 v_1 & -\beta_2^2 v_2 & -\beta_3^2 v_3 \end{pmatrix}.$$

If the matrix V has full row rank, then price regulation can Pareto improve the competitive equilibrium allocation. In this case, any change in the individuals' utility can be achieved, in particular a change that improves the utility of both.

If the ratios of the marginal utilities of income of the individuals are not the same across all states of the world, $\beta_1^1/\beta_1^2 \neq \beta_2^1/\beta_2^2$ or $\beta_3^1/\beta_3^2 \neq \beta_2^1/\beta_2^2$, for the matrix V to have full row rank it is sufficient that $v_s \neq 0$, for every state of the world. Since $v_s = 0$ if and only if $e_{1,s}^1/e_{1,s}^2 = \gamma_s^1/\gamma_s^2$, generically in the endowments of individuals it is possible to Pareto improve on the competitive allocation.[2]

For the two individuals case, the net supply of individual i equals the net demand of individual i' at a competitive equilibrium. Typically, because of market incompleteness, individuals have a different marginal utility of income in a particular state. This implies a full rank of the matrix V, and thereby the possibility of Pareto improving price regulations. For cases with more individuals, the same logic applies. Since marginal utilities of income are different because of market incompleteness, the matrix V is typically of full row rank, and Pareto improving price regulations are possible. The only caveat is that the argument only applies when indirect utility functions are differentiable at competitive equilibrium prices.

Remarks

(1) The example is robust against perturbations in endowments and the parameters of the quasi-linear utility indices that satisfy the restrictions we imposed. It is also robust against perturbations of the logarithmic term.

(2) Generalizations of the example, to allow, for instance, for more individuals, are possible, but not of interest if restricted to allow for explicit computations.

(3) The quasi-linearity of the cardinal utility eliminates income effects. Prompted by the intuition behind the result in Geanakoplos and Polemarchakis (1986), one may

[2] For the specification of parameters given in footnote 1,

$$V = \begin{pmatrix} 0 & -\frac{1}{3} & \frac{1}{6} \\ 0 & \frac{1}{6} & -\frac{1}{3} \end{pmatrix}.$$

Both individuals benefit if the price of commodity 1 in states 2 and 3 is fixed below its competitive equilibrium value.

find this surprising; however, it is not. When reallocations of assets are the policy instrument, income effects are essential: in the absence of differences in income effects across individuals, reallocations of portfolios do not affect prices in spot markets and constrained optimality obtains. Here, variations in spot commodity prices are themselves the policy instrument and income effects are not required for Pareto improvements.

(4) With multiple fix-price equilibria in the neighborhood of a competitive equilibrium, local existence of a fix-price equilibrium near a competitive equilibrium may fail, and a robust example without the possibility of Pareto improving price regulation might be constructed.

(5) In the example, with quasi-linear cardinal utility indices, the Pareto improving deviation of prices from their competitive equilibrium values cannot be chosen independently of the state of the world; with differentiably strictly concave cardinal utility it can,[3] as long as the number of commodities exceeds the number of individuals.

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[3] Herings and Polemarchakis (2005); John Geanakoplos insisted on this point.