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Intertemporal market division: A case of alternating monopoly

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Abstract

In this paper, we report on an equilibrium with market dominance that exists in a simple two-firm model that features neither entry barriers nor sophisticated punishment strategies. This equilibrium induces an intertemporal market division in which the two firms alternate as monopolists – despite the fact that the model also sustains a Cournot duopoly. Even when initially both firms are active in the market, the alternating monopoly reveals itself rather quickly. Moreover, it Pareto dominates the Cournot equilibrium – as it is close to the cartel outcome. Several examples of what well may be such alternating monopolies are presented.

JEL classification: C73; D43; L13

Keywords: Dynamic competition; Oligopolistic competition; Stochastic games; Monopoly.

1. Introduction

To escape the discipline of competitive markets, even when only partially or temporarily, typically returns positive economic profits. As a result, firms are tempted to seek anticompetitive arrangements in order to do so. Many examples of how firms may be able to collude have been forwarded in the literate, ranging from overt cartels that divide the product market in regular conspicuous meetings, to tacit geographical market divisions that rely on ingeniously hidden basing point systems. A potential form of sharing markets that has so far received little attention is intertemporal market division, in which firms avoid direct competition by taking turns in serving the market for certain periods of time. There is, however, quite some indication that this type of collusion is

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indeed practiced. It has been observed, for example, that large companies of relatively homogeneous commodities that are only substantially differentiated by application of marketing methods, such as cola, tend to alternate their advertising campaigns. Dixit and Nalebuff (1991), e.g., refer to a report on "60 Minutes" that in a span of 52 weeks Coca Cola and Pepsi Cola each offered 26 weeks of price promotions between which there was not one single overlap. They calculate the probability that this would occur by luck if the two companies were acting independently and each offered 26 weeks of "couponing" as 1/495,918,532,948,104, which they conclude is strong statistical evidence for collusive behavior (Dixit and Nalebuff, p. 193).

Likewise, television stations often intertemporally alternate their prime shows, such as news services and sit-coms, so that they do not compete for viewers in the same time slot. Also, summer events, like open air pop festivals, or local fancy fairs, are planned in different weeks of the season. And similarly, major sports games, particularly important matches such as finals, are planned not to overlap – both within one and the same sport, and among different sports. A final example, that involves the direct production of physical commodities, can be found in the market for video games. There, major players, Nintendo and Sony, tend to alternate the introduction dates of their next respective video consoles, the brand specific hardware on which to run their uniquely compatible games – in which another alternating introduction cycle can be observed, albeit one with a much shorter longitude.[1]

In this paper, we report on the existence of a Markov perfect equilibrium that induces an intertemporal division of the market in the simplest possible dynamic game of competition with entry and exit. It features only two players, so that the intertemporal market division takes the form of an alternating monopoly. Incumbents are in Cournot competition, entry and exit decisions are mixed strategies, and there is perfect and complete information. Although the model also has a long-run Cournot duopoly equilibrium, the probability that an industry structure of alternating monopoly is observed converges quickly to one, for all reasonable discount factors, when the firms coordinate on alternating. Moreover, the alternating monopoly equilibrium Pareto dominates the Cournot equilibrium. That is, both firms are able to obtain higher profits by dividing the market intertemporally between them.

The endogenous possibility of alternating monopoly we point out has, to the best of our knowledge, not been reported in the literature so far. Closest, probably, comes an equilibrium found in Maskin and Tirole (1988), in which firms also take turns operating as a monopolist (Maskin and Tirole, pp. 564–565). This equilibrium is, however, importantly different from ours, as the authors consider an alternating-move model, whereas in our model firms act simultaneously. As a result, their equilibrium is not Markovian (and otherwise not symmetric) – which the authors regret, as they advocate Markov perfect equilibrium there and elsewhere, e.g., Maskin and Tirole (2001). Davies (1991) also features exogenous alternation in strategies, to find that when costs are asymmetric, the market structure will converge to a natural monopoly with the most efficient firm as the sole producer. Moreover, due to the alternation of moves in her model, alternating monopoly cannot be a Markov-perfect equilibrium

[1] Cf. Kent (2001).

(Davies, 1991, p. 526, footnote 13). The simultaneity of decisions in our model, we believe, presses the analysis as more fitting to questions of entry and exit, and the equilibrium found as all the more remarkable.

The equilibrium we find is surprising indeed, for it reveals a possible failing of potential competition, without any of the known arguments for it applying. The benchmark model in the central debate in industrial economics on the question whether or not the threat of potential competition disciplines already incumbent firms is that of perfect contestability, definitely laid out in Baumol et al. (1982). The model predicts that, in the absence of sunk costs, potential competition is sufficient to guarantee pricing at marginal costs, even when the number of both incumbents and potential entrants is low. Traditionally, going back to Bain (1956), contributions that seek to negate this result, and instead show that stable positions of market power for incumbent firms are possible, rely on the presence of substantial sunk costs to throw up barriers to entry.

In a more sophisticated argument, in Stiglitz (1981) and Dasgupta and Stiglitz (1988), Cournot competition on quantities among potential entrants that simultaneously make their entry decisions, serves as a form of 'outside' competition among the potential entrants that discourages entry for each one of them. This, in fact, fortifies the position of incumbent firms when the number of potential entrants increases. When entry is played as a mixed strategy, however, as in Dixit and Shapiro (1986), or when it is sequential, as in Vives (1988) and Waldman (1991), potential competition generally again increases actual competition and hence welfare. This is counterbalanced by a literature that followed up on an example of socially excessive entry in the presence of sunk costs by Mankiw and Whinston (1986), including a very general setup in Amir and Lambson (2003).

In a literature that answered Selten's chain store paradox, see Selten (1978), credible deterrence of entry relies, e.g., on the presence of imperfect information that allows for building a reputation to fight entry. Likewise, through sophisticated punishment strategies of various kinds, dominant positions can be sustained in repeated games of entry.

The common perception that potential competition helps actual competition, therefore, relies on the imperfections that drive the above results being quite specific.[2] Yet, our basic model is perfect in all these respects: there are no sunk costs of production, entry or exit, so that there are no barriers to entry; incumbents – when there are more than one – compete; entry and exit decisions take the form of mixed strategies; players are perfectly and completely informed; and, by limiting the analysis to Markov strategies, which are independent of history and time, we rule out collusion sustained by 'tailor made' punishment. Therefore, one would expect that the model would quickly display a long-run Cournot duopoly for the case of a market with two potential entrants. As said, this is a possible equilibrium indeed. Yet, we find that the alternating monopoly equilibrium is dominant.

That fact that our model features no, or relatively small entry and exit costs – with the latter our analysis can deal as well, as set out in Section 5 – makes that it fits the examples given above of tacitly collusive production alternation quite well. In none of

[2] Cf. Audretsch et al. (2001), as well as Amir and Lambson (2000).

these examples the firms involved exit in the sense that they fold up their tents and leave, to set up again the day after tomorrow. They remain in the market, yet ‘lay low’ in turns for some periods, to the benefit of their competitors. Therefore, the costs of switching from producing to not producing are small, and there are no serious sunk costs from existing either. What is crucial, however – and necessary for an alternating monopoly equilibrium to exist – is that this ‘laying low’ is credible, in that a cola producer without an advertising campaign readily available would need some time to design and produce one, and so would a video game console manufacturer. Likewise, setting up a rival open air pop event is not done overnight either. Production involves a start-up lag, that is. Many markets seem to have these, or similar characteristics, so that, next to the known types of market division, intertemporal market division may well be a form of collusion that reduces competition, and thus welfare.

The remainder of this paper is organized as follows. Section 2 introduces our simple dynamic model with entry and exit. Section 3 establishes the existence of a Markov perfect equilibrium in which the two firms divide the market intertemporarily. In Section 4 it is shown that the alternating monopoly is likely, in the sense that the probability that an intertemporal market division is observed converges to one over time. By introducing positive exit and entry costs, as well as fixed costs, in Section 5 the robustness of the alternating monopoly equilibrium is studied. It is found that for modest levels of these costs, the equilibrium stays to exist. Section 6 closes with some concluding thoughts.

2. A simple dynamic entry-and-exit model

Consider the simplest of possible dynamic models of entry and exit. Two firms, producing with constant marginal costs c and no fixed costs, intend to serve a market characterized by the a linear demand curve, such that the market price p depends on the quantity totally produced Q as

$$p = a - bQ. \tag{1}$$

For each firm, the decision to be active in the market comes up every period. That is, when a firm is in the market, it decides each period first how much to produce, and second whether to remain in the market for another period or to exit. When it is out of the market it decides whether to enter in the next period or not. Firms take these decisions simultaneously and independently of one another, and they can randomize over their strategies. As a consequence, no firm can observe the decisions of the other. In the following, there are neither entry, nor exit costs.

At any given time, the industry structure is one of four possible structures: either both firms are active in the market, or neither of them – so that there is no production at all – or one firm is active and the other is not, or vice versa. Each transition from a given market structure to another from one period to the next depends on the simultaneous and independent decisions of the firms whether or not to be active for production in the next period. Consequently, this dynamic entry-and-exit model can be modeled as a stochastic game, where the only stochastic element is caused by the potentially probabilistic entry and exit behavior of the firms.

The most general strategy a firm can formulate to play this game is a behavior strategy. In that, each firm specifies for each period, each market structure at that period, and each history leading up to it with which probability to be active in production next period. To this end, it is assumed that in each period both firms know the history of the market structure up to and including the current structure, and that this is common knowledge. Furthermore, in formulating their behavior strategies, firms discount the future profits they foresee by a factor $\delta \in (0, 1)$.

A Nash equilibrium for this model is a pair of behavior strategies, one for each firm, for which it holds that each firm's strategy is a best response to the strategic decision of its opponent. In the following, we concentrate on Markov strategies. These are behavior strategies that are history and time independent, so that at all times actions depend only on the current state.[3] A Markov perfect equilibrium for this model is a pair of Markov strategies that yields a Nash equilibrium in every proper subgame. Notice that the state contains only information on the current market structure – which firm is in and which is out – and excludes any information on past behavior. As the two firms are really identical – except perhaps for their position in the market – we consider symmetric Markov perfect equilibria. That is, a firm behaves in equilibrium as the other firm would, when their positions were reversed.

Several motivations for analyzing Markov perfect equilibria in the set of Nash equilibria can be found in Maskin and Tirole (2001). Markov strategies describe the simplest form of behavior that is consistent with rationality, enable a clean and unobstructed analysis of the influence of state variables, and substantially reduce the number of parameters so that they can be conveniently simulated. Furthermore, Markov strategies captures the idea that 'bygones are bygones' more completely than does the concept of subgame-perfect equilibrium. As a consequence, Markov perfection reduces the multiplicity of equilibria that the Folk Theorem supports.[4] This latter motivation is of principal interest here: Restriction to Markov strategies, coupled with our state space, excludes the possibility to tailor behavior in an attempt to punish the rival firm, should it break the alternating equilibrium.

In the simple dynamic model considered here, in which the entry decisions of the potential rivals cannot yet be observed when deciding on the production level and potential entrants are not able to produce instantaneously upon entry, in equilibrium all active firms produce the static Cournot quantity. Hence, when there are k firms active in the market (here, k equals 1 or 2), each of the k firms produces the static Cournot quantity given by

$$q_k^C = \frac{(a - c)}{(k + 1)b}, \tag{2}$$

[3] In general, Markov strategies can depend on time. Here, however, we resort to the definition of Fudenberg and Tirole, where they do not when the action spaces, instantaneous profits and the transition dynamics are time-invariant – Cf. Fudenberg and Tirole (1995, p. 505). These strategies are also known as stationary strategies.

[4] In fact, the number of Markov perfect equilibria in the type of model considered here can be shown to be generically finite, see Haller and Lagunoff (2000), and even generically odd, see Herings and Peeters (2003).

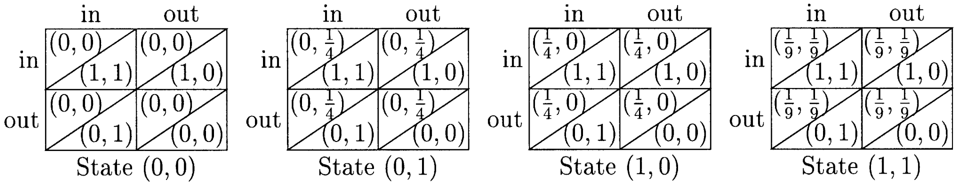

Fig. 1. The dynamic entry-and-exit model as a stochastic game.

and the instantaneous profit to each of the k active firms is

$$\pi_k^C = \frac{(a-c)^2}{(k+1)^2 b}. \tag{3}$$

In Fig. 1 this stochastic game is depicted, where it is assumed, without loss of generality, that $(a-c)^2/b = 1$. This can simply be justified by the choice of units. A firm choosing the strategy 'in' will produce in the next period, and a firm choosing strategy 'out' will be out of production in the next period. The instantaneous profits of the firms are given by the tuple in the upper-left part of the cells. The lower-right part of the cell indicates the next state, reached with probability one – $(1,1)$ means for example both firms in, and $(0,1)$ firm 1 out, firm 2 in.

3. Alternating monopoly

Conventional wisdom quickly leads to the claim that the market dynamics modeled above will typically result in the two firms sharing the market forever. After all, in the Cournot duopoly situation, there is a substantial profit over and above the entry costs – here assumed to be zero – for each firm. Moreover, there is no dynamic limit pricing – which would be fruitless in the absence of barriers to entry – nor do firms deploy sophisticated punishment strategies – as they cannot by construction. And indeed it is the case that 'always stay when active' and 'always enter when not active' – or, alternatively, 'produce in the next period no matter what the current market structure is', is a Markov perfect equilibria. We refer to the appendix for an explicit characterization of the set of Markov perfect equilibria. This characterization enables the reader to verify easily where a given strategy profile constitutes a Markov perfect equilibrium or not.

Table 1 shows, for a discount factor $\delta = 0.95$, the equilibrium strategies for each firm in each state, as well as the expected total discounted profits to the firms by coordinating on this equilibrium for each possible initial state.

Irrespective of whether initially neither firm, both firms, or one of the two firms is in the market, each will produce in the next period with probability 1. There is a slightly higher payoff for a firm that happens to be the first incumbent, enjoying an instantaneous monopoly profit for a single period, when the initial market structure featured a monopoly.

Apart from this known equilibrium, however, a different Markov perfect equilibrium turns out to exist, with a fundamentally different nature. Again for $\delta = 0.95$, Table 2 displays its diagnostics.

Table 1
Markov perfect equilibrium with two active firms

(Initial) state	Equilibrium strategies				Equilibrium	
	Firm 1		Firm 2		Payoffs	
	In	Out	In	Out	Firm 1	Firm 2
(0,0)	1.0000	0.0000	1.0000	0.0000	2.1111	2.1111
(0,1)	1.0000	0.0000	1.0000	0.0000	2.1111	2.3611
(1,0)	1.0000	0.0000	1.0000	0.0000	2.3611	2.1111
(1,1)	1.0000	0.0000	1.0000	0.0000	2.2222	2.2222

Table 2
Markov perfect equilibrium with alternating monopoly

(Initial) state	Equilibrium strategies				Equilibrium	
	Firm 1		Firm 2		Payoffs	
	In	Out	In	Out	Firm 1	Firm 2
(0,0)	0.9306	0.0694	0.9306	0.0694	2.3055	2.3055
(0,1)	1.0000	0.0000	0.0000	1.0000	2.4359	2.5641
(1,0)	0.0000	1.0000	1.0000	0.0000	2.5641	2.4359
(1,1)	0.9306	0.0694	0.9306	0.0694	2.4166	2.4166

In this equilibrium, when none of the firms produces today, each will produce tomorrow with probability 0.9306.[5] This means that with probability $(1-0.9306)^2=0.0048$ no firm will enter, with probability 0.1292 exactly one – the probabilities are equally split over the two possible situations – and with probability 0.8660 both firms will be active tomorrow. Similarly, when in a certain period both firms are active, the next period all market structures can be reached with positive probability. However, once a firm is the only firm in the market today it will with certainty stop producing tomorrow, whereas the opponent firm that is not in the market today will be for sure one period later. The market will, in other words, display an alternating monopoly.

Once identified as a candidate equilibrium of the model, it is relatively straightforward to check that the alternating monopoly equilibrium is indeed a Markov perfect equilibrium. For that, we explain in the appendix that only unilateral one-stage deviations are relevant to consider. To see next that one-stage deviations from the alternating monopoly equilibrium strategies are not profitable, the following. Obviously, it does not pay to not take one's turn to enter when the other firm exits. Suppose instead a firm considers not leaving the market when it is the other firm's turn to serve it. Then, in the next period the market will display Cournot duopoly, which indeed gives

[5] To be more precise, this probability is equal to $(-287+\sqrt{127969})/76$, but in the following we will work with the more convenient decimal approximation. An explicit derivation is given in the appendix.

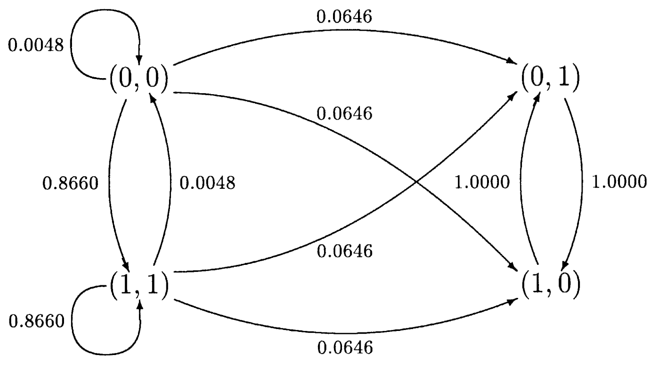

Fig. 2. Transition probabilities in market structure.

the defector a one-shot pay-off increase. However, with high probability the market is expected to stay in Cournot competition for a number of periods, rather than just one. This so, since the equilibrium exit probabilities associated with two firms present in the market are close to zero. As a result, total discounted profits are lower, so that firms will refrain from deviating from the alternating monopoly equilibrium. For a more careful making of these arguments, see the appendix.

4. Is an alternating monopoly likely?

Having established its existence, the natural question to ask is how likely and robust the alternating monopoly equilibrium is. One approach to this question is to consider how long it takes before an alternating monopoly reveals itself, when both firms coordinate on that equilibrium. A first observation towards this is on the role of the discount factor δ. For values of δ large enough, the alternating monopoly equilibrium exists and can be calculated. However, when firms become sufficiently impatient, it no longer pays to divide the market intertemporally, and the equilibrium disappears. This critical value of δ is equal to 0.80, which corresponds to an interest rate of 25% per period. For all higher discount factors (lower interest rates), an equilibrium industry structure characterized by an alternating monopoly can be found.

The likelihood of indeed observing an alternating monopoly can subsequently be studied as follows. Once the industry is in an alternating monopoly situation in a certain period, it will stay in that equilibrium forever – where at even periods the one firm is active and at odd periods the other. When the market structure is not a monopoly in a certain period, there is a positive probability that it will end up in a monopoly next period. Consequently, the cumulative probability of an alternating monopoly increases in time.

To see the speed with which this likelihood converges to one, consider the Markov process induced by the alternating monopoly equilibrium. Its transition probabilities are illustrated graphically in Fig. 2 – again for $\delta = 0.95$. These state-transitions can be

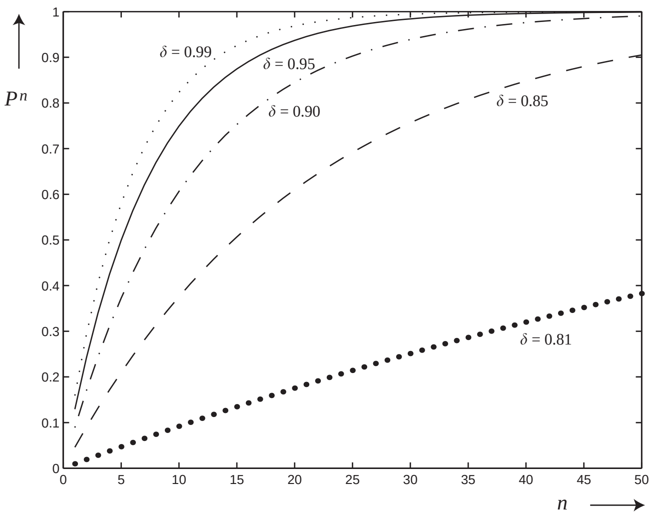

Fig. 3. Speed of convergence to alternating monopoly for different discount factors.

caught in the transition-matrix P defined as

$$P = \begin{bmatrix} 0.0048 & 0.0646 & 0.0646 & 0.8660 \\ 0.0000 & 0.0000 & 1.0000 & 0.0000 \\ 0.0000 & 1.0000 & 0.0000 & 0.0000 \\ 0.0048 & 0.0646 & 0.0646 & 0.8660 \end{bmatrix}.$$

The probability of having a certain market structure after two periods knowing the current market structure is given in the matrix P^2. When the current market structure is ω, the probability that the market structure is $\bar{\omega}$ exactly n periods later is given by the value of the matrix cell in row ω and column $\bar{\omega}$ of the matrix P^n. Regardless of what market structure is initially prevailing, therefore, the probability that an alternating monopoly is found after n periods is at least equal to $P^n(1,2) + P^n(1,3)$.[6]

For all periods up to 50 we have computed these probabilities. The resulting data are displayed in Fig. 3, for various values of δ. Clearly, except for values of δ close to 0.80, within some 50 periods the probability of an alternating monopoly rapidly converges to one. In fact, for $\delta = 0.90$ the cumulative probability of an alternating monopoly is larger than 0.99 after exactly 50 periods. For $\delta = 0.95$ this is the case after 34 periods, for $\delta = 0.99$ after 27 periods, but even when $\delta = 0.85$, it takes 98 periods for the cumulative probability of observing alternating monopoly to reach 0.99.

There remains the question how reasonable it is to assume that coordination on the alternating monopoly equilibrium will happen. Towards answering this, it should

[6] Naturally, it is also at least equal to $P^n(4,2) + P^n(4,3)$. Moreover, it can be derived that $P(1,2) = P(1,3) = P(4,2) = P(4,3)$.

be noted that in any game with multiple equilibria a similar question arises for each particular equilibrium. Hence, for that matter, the alternating monopoly case is no less likely than the Cournot equilibrium. However, if anything, the alternating monopoly equilibrium is strictly more appealing to both firms, as they each make more profits, irrespective of the initial market structure. In fact, firms are able to extract close to the cartel profits, whereas a cartel is not sustainable as an equilibrium in this model.

In the case reported on above, where $\delta = 0.95$, the total discounted profits of each firm in the alternating monopoly equilibrium is at least 8.5% higher than that had in the Cournot equilibrium. This is when both firms are initially in the market – compare to this end Tables 1 and 2. The firm that is first in the market in the initial states with one firm so raises its payoff 15.5% over the Cournot profit. On average, the alternating monopoly equilibrium generates a discounted profit that is almost 12% higher than Cournot profits. For lower values of δ this difference becomes smaller, and it eventually converges to zero when δ gets very close to 0.80. That is, for all values for which the alternating monopoly equilibrium exists, it is strictly payoff dominant.

5. Alternating monopoly with entry, exit and fixed costs

The simplicity of the model strengthens the surprise of our result, we believe. Whereas in the existing literature potential anticompetitive effects typically derive from the presence of barriers to entry, in the present analysis competition fails despite of the fact that firms can exit, (re)enter and produce without incurring fixed costs. Yet, it is of interest to consider whether the analysis is robust against the introduction of such costs. Also, one could argue that the examples of alternating cola advertising campaigns, the releases of new computer game consoles and seasonal events discussed in the introduction do feature modest levels such costs and would be better explained with them included. As it turns out, however, our analysis can deal with either type of costs just fine.

It is true that, since (re-)entry costs are to be incurred time and again in the alternating monopoly equilibrium, sufficiently high costs of entry may be incompatible with an alternating monopoly equilibrium, for they outweigh the gain of serving the market in turn. However, for $\delta = 0.95$, for example, modest levels of entry costs of almost 8.5% of the instantaneous monopoly profits still sustain the alternating monopoly equilibrium as a profitable opportunity.[7] With higher (re-)entry costs the alternating monopoly equilibrium no longer exists, leaving the Cournot equilibrium. This leads to the counterintuitive conclusion that sufficiently high – but not too high – barriers to entry can, in fact, support a relatively competitive environment. For all entry costs for which the alternating monopoly equilibrium exists, however, the latter is strictly payoff dominant.

The introduction of fixed costs, made in every period when producing, does not jeopardize our results either. Yet, it does drive a wedge between the understandings of exit/entry as 'folding up the tent' and 'laying low', discussed in the introduction,

[7] That is, entry costs up to 0.0208 which is 8.33% of normalized profits – which are equal to 0.25.

so that our model applies to both. That is, the alternating monopoly equilibrium exists for various non-specifically chosen parameter values.[8]

In fact, as fixed costs are incurred in either type of equilibrium, alternating monopoly or Cournot, they have no specific bearing on the issue at hand. Fixed costs do, however, open possibilities for interesting further research. We find, for example, specifications for which the Cournot equilibrium involves losses, whereas the alternating monopoly equilibrium is still sustainable – this is the case when discount factors and fixed costs are sufficiently high. The reason is that monopolists bear the fixed costs only every second period. The theory of natural monopoly seems, in other words, compatible with firms that share the market intertemporally.

6. Concluding thoughts

We have reported on the possibility, so far unknown, that in the simplest of dynamic, simultaneous-move models of entry and exit, where none of the traditional assumptions used to show sustainable dominant positions, such as barriers to entry, simultaneous entry, retaliation strategies, or alternating moves are made, an alternating monopoly arises. Moreover, the payoffs involved in the alternating monopoly approach the cartel outcome, despite the fact that the market allows two firms to make profit when active, and does not sustain a cartel. Consequently also, consumer welfare is importantly harmed by the ability of firms to tacitly collude on this intertemporal market division. Production is decreased below, and prices increased above their duopoly levels, resulting in an increase in dead-weight loss.[9] Hence, antitrust authorities should be wary of this form of tacit collusion.

This is all the more so, since intertemporal market division may come in many disguises. For the two-firm model analyzed here, a period-two cycle as observed in the cola and the game console market reveals itself unmistakenly over time. Yet, the same may not be true for intertemporal market divisions with a larger number of potential entrants. An example of such a less obvious collusive equilibrium may be the market for taxi rides at major hubs, such as train stations and airports. There, common understanding often is that newly arriving taxi's that just completed a fare queue up and await their turn for the next ride. Often this queuing is facilitated by the airport infrastructure – such as a narrow passage, set off with curb stones, that does not allow overtaking – that prevents waiting caps to skip the line, or even a regulator whose job it is to point customers to their designated taxi. It provides a credible 'laying low'

[8] In fact, the alternating monopoly equilibrium will exist for all triples of δ, e and f for which the probability by which each firm decides to either enter when the state is $(0,0)$, or stay in the market when the state is $(1,1)$ is between zero and one. This probability is given by

$$\sigma = \frac{-(5\delta + 4\delta^3 - 36\delta^2 e - 36\delta^3 f) + \sqrt{(5\delta + 4\delta^3 - 36\delta^2 e - 36\delta^3 f)^2 - 4(5\delta^2 - 4\delta^3 + 36\delta^2 e + 36\delta^3 f)(-9\delta + 36e + 36\delta f)}}{2(5\delta^2 - 4\delta^3 + 36\delta^2 e + 36\delta^3 f)},$$

where e and f are the entry costs and fixed costs respectively.

[9] In fact, since in the linear model under consideration dead-weight loss is half of the total (instantaneous) profits between the market parties, the discounted welfare loss left in alternating monopoly is also almost 12% higher on average than that in Cournot equilibrium.

commitment of the potential entrants. As a consequence, the first taxi in the queue is in a temporary monopoly position, and fares are likely to be priced excessively.

An n-firm extension of the present analysis would accordingly stretch the period cycle. When the number of firms increases, an alternating monopoly equilibrium would imply $n-1$ periods of being inactive for each firm, awaiting its turn to dominate the market every nth time. Such an equilibrium could be Markov perfect and simultaneously symmetric, but not anonymous. It would require the behavior of firms to depend on the identity of the incumbents. Even anonymous, symmetric Markov perfect equilibria inducing collusion may still exist: Firms enter with small probability when being inactive and exit with high probability when active, such that all firms act as a monopolist once in a while. Obviously, this puts the patience of firms to the test. But, as long as the discount factor is large enough, all firms will obtain a higher payoff by being a monopolist every nth period than being active all periods with all firms present. Since the static Cournot profit decreases quadratically in the number of firms present, the difference between the profits in the two situations increases, so that the collusive equilibrium may become more likely as the number of firms increases. It is also very well possible that a different type of equilibrium exists in an n-firm version of our model, namely one in which clusters of firms tacitly collude on sharing the market each period with only a restricted number of them. Over time, these clusters then alternate, again increasing profits at the expense of welfare. The existence and likelihood of these equilibria are subject of further research.

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Appendix

In the following, a general characterization of the set of Markov perfect equilibria for stochastic games by means of a formulation of a system of equations and inequalities is presented. Moreover, the alternating monopoly equilibrium that is displayed in Table 2 is analytically determined and shown indeed to be a Markov perfect equilibrium.

In general, a finite discounted stochastic game is given by the tuple

$$\Gamma = \langle N, \Omega, \{S^i_\omega\}_{(i,\omega)\in N\times\Omega}, \{u^i\}_{i\in N}, \pi, \delta\rangle,$$

in which N denotes the finite set of players, Ω the finite set of states, and S^i_ω is the finite set of actions that player i has at its disposal in state $\omega \in \Omega$. The instantaneous payoff to player i in state ω when the players play $s_\omega = (s^i_\omega)_{i\in N}$ is given by $u^i(\omega, s_\omega)$. The probability of going from state ω to state $\bar{\omega}$ when the players play s_ω is given by $\pi(\bar{\omega}|\omega, s_\omega)$. The players are allowed to randomize their actions. Their instantaneous payoff-function is extended to satisfy the expected utility property. The transition mapping is extended similarly. Players maximize the total stream of expected payoffs, where the future payoffs are discounted by discount factor $\delta \in (0,1)$.

The game proceeds as follows. Each player i selects at the initial state ω^0 an action $\sigma^i_{\omega^0} \in \Delta(S^i_{\omega^0})$. Then two things happen, both depending on the current state ω^0 and the action choices $\sigma_{\omega^0} = (\sigma^i_{\omega^0})_{i \in N}$:

(1) player i earns $u^i(\omega^0, \sigma_{\omega^0})$, and
(2) the system jumps to the next state ω^1 according to the outcome of the chance experiment given by $\pi(\cdot|\omega^0, \sigma_{\omega^0})$.

Subsequently, in the next period, all players are informed about the previous actions chosen by the players, and of the new state ω^1. In this next period, the above procedure is repeated, starting from the state ω^1.

It follows from Sobel (1971), see also Theorem 3.5 in Herings and Peeters (2003), that a stationary strategy-tuple σ is a Markov perfect equilibrium if and only if it is part of a solution to

$$u^i(\omega, \sigma^{-i}_\omega, s^i_{\omega j}) + \delta \sum_{\bar{\omega} \in \Omega} \pi(\bar{\omega}|\omega, \sigma^{-i}_\omega, s^i_{\omega j}) \mu^i_{\bar{\omega}} + \lambda^i_{\omega j} - \mu^i_\omega = 0$$

$$(s^i_{\omega j} \in S^i_\omega, \omega \in \Omega, i \in N),$$

$$\lambda^i_{\omega j} \geqslant 0, \quad \sigma^i_{\omega j} \geqslant 0, \quad \lambda^i_{\omega j}\sigma^i_{\omega j} = 0 \quad (s^i_{\omega j} \in S^i_\omega, \omega \in \Omega, i \in N),$$

$$\sum_{s^i_{\omega j} \in S^i_\omega} \sigma^i_{\omega j} - 1 = 0 \quad (\omega \in \Omega, i \in N).$$

Here, $\lambda^i_{\omega j}$ is the shadowprice of playing action $s^i_{\omega j}$, i.e., the disutility of a marginal increase in the probability $\sigma^i_{\omega j}$ by which pure action $s^i_{\omega j}$ is played at the initial period, and μ^i_ω is the expected payoff of player i when the initial state is ω, σ^{-i} is played by its opponents, and player i chooses a best response against σ^{-i}. The system above suggests that only one-shot deviations have to be considered. This is indeed the case, for the following reason.

Suppose all other players play stationary strategies σ^{-i}, to which σ^i is the best stationary response for player i. If player i is not able to improve its expected payoff by a deviation from its strategy σ^i in one stage only, then it follows by a backward induction argument, that neither will it by any finite number of deviations from its strategy. Infinitely many changes will also not improve player i's expected payoff. Suppose that it would. Then, by a profit-to-go argument, player i would also be able to increase its payoff by finitely many changes, which is not possible.[10]

[10] Define $M = \max_{(i,\omega,s_\omega)} |u^i(\omega, s_\omega)|$. Then the maximum payoff a player can earn from time k onwards is bounded from above by $\delta^k(1 + \delta + \delta^2 + \cdots)M = \delta^k M/(1-\delta)$, the so-called maximum 'profit-to-go' value. Suppose player i is able to improve its payoff by ϵ through infinitely many changes. When k grows large, the profit-to-go value is at a certain point less than ϵ (this is when $k > \log(\epsilon(1-\delta)/M)/\log(\delta)$). This means that the payoff improvement by changes until time k (finitely many changes) was positive.

Searching for equilibria where the two firms behave symmetrically, the first line of the system of (in)equalities becomes

$$
\begin{aligned}
s^1_{(0,0)\text{out}}: &\quad 0+\delta\{\sigma^1_{(0,0)\text{out}}\mu^1_{(0,0)}+\sigma^1_{(0,0)\text{in}}\mu^1_{(0,1)}\}+\lambda^1_{(0,0)\text{out}}=\mu^1_{(0,0)},\\
s^1_{(0,0)\text{in}}: &\quad 0+\delta\{\sigma^1_{(0,0)\text{out}}\mu^1_{(1,0)}+\sigma^1_{(0,0)\text{in}}\mu^1_{(1,1)}\}+\lambda^1_{(0,0)\text{in}}=\mu^1_{(0,0)},\\
s^1_{(0,1)\text{out}}: &\quad 0+\delta\{\sigma^1_{(1,0)\text{out}}\mu^1_{(0,0)}+\sigma^1_{(1,0)\text{in}}\mu^1_{(0,1)}\}+\lambda^1_{(0,1)\text{out}}=\mu^1_{(0,1)},\\
s^1_{(0,1)\text{in}}: &\quad 0+\delta\{\sigma^1_{(1,0)\text{out}}\mu^1_{(1,0)}+\sigma^1_{(1,0)\text{in}}\mu^1_{(1,1)}\}+\lambda^1_{(0,1)\text{in}}=\mu^1_{(0,1)},\\
s^1_{(1,0)\text{out}}: &\quad \frac{1}{4}+\delta\{\sigma^1_{(0,1)\text{out}}\mu^1_{(0,0)}+\sigma^1_{(0,1)\text{in}}\mu^1_{(0,1)}\}+\lambda^1_{(1,0)\text{out}}=\mu^1_{(1,0)},\\
s^1_{(1,0)\text{in}}: &\quad \frac{1}{4}+\delta\{\sigma^1_{(0,1)\text{out}}\mu^1_{(1,0)}+\sigma^1_{(0,1)\text{in}}\mu^1_{(1,1)}\}+\lambda^1_{(1,0)\text{in}}=\mu^1_{(1,0)},\\
s^1_{(1,1)\text{out}}: &\quad \frac{1}{9}+\delta\{\sigma^1_{(1,1)\text{out}}\mu^1_{(0,0)}+\sigma^1_{(1,1)\text{in}}\mu^1_{(0,1)}\}+\lambda^1_{(1,1)\text{out}}=\mu^1_{(1,1)},\\
s^1_{(1,1)\text{in}}: &\quad \frac{1}{9}+\delta\{\sigma^1_{(1,1)\text{out}}\mu^1_{(1,0)}+\sigma^1_{(1,1)\text{in}}\mu^1_{(1,1)}\}+\lambda^1_{(1,1)\text{in}}=\mu^1_{(1,1)},
\end{aligned}
$$

where the game data and variables are written from firm 1's perspective, and firm 2 behaves identically by the symmetry assumption. In the alternating monopoly, the probability that firm 1 enters the market in state $(0,1)$ and exits in state $(1,0)$ equals one: $\sigma^1_{(0,1)\text{in}}=\sigma^1_{(1,0)\text{out}}$. Moreover, $\mu^1_{(0,1)}=\frac{1}{4}(\delta+\delta^3+\delta^5+\cdots)=\frac{1}{4}\delta/(1-\delta^2)$ and $\mu^1_{(1,0)}=\frac{1}{4}(1+\delta^2+\delta^4+\cdots)=\frac{1}{4}1/(1-\delta^2)$. In state $(0,0)$ and $(1,1)$ the firm plays mixed, such that by complementarity the corresponding λ's are equal to zero. Substitution of these data simplifies the system of equations to:

$$
\begin{aligned}
s^1_{(0,0)\text{out}}: &\quad 0+\delta\left\{\sigma^1_{(0,0)\text{out}}\mu^1_{(0,0)}+\sigma^1_{(0,0)\text{in}}\frac{1}{4}\frac{\delta}{1-\delta^2}\right\} &&+0 &&=\mu^1_{(0,0)},\\
s^1_{(0,0)\text{in}}: &\quad 0+\delta\left\{\sigma^1_{(0,0)\text{out}}\frac{1}{4}\frac{1}{1-\delta^2}+\sigma^1_{(0,0)\text{in}}\mu^1_{(1,1)}\right\} &&+0 &&=\mu^1_{(0,0)},\\
s^1_{(0,1)\text{out}}: &\quad 0+\delta\{\mu^1_{(0,0)}\} &&+\lambda^1_{(0,1)\text{out}} &&=\frac{1}{4}\frac{\delta}{1-\delta^2},\\
s^1_{(0,1)\text{in}}: &\quad 0+\delta\left\{\frac{1}{4}\frac{1}{1-\delta^2}\right\} &&+0 &&=\frac{1}{4}\frac{\delta}{1-\delta^2},\\
s^1_{(1,0)\text{out}}: &\quad \frac{1}{4}+\delta\left\{\frac{1}{4}\frac{\delta}{1-\delta^2}\right\} &&+0 &&=\frac{1}{4}\frac{1}{1-\delta^2},\\
s^1_{(1,0)\text{in}}: &\quad \frac{1}{4}+\delta\{\mu^1_{(1,1)}\} &&+\lambda^1_{(1,0)\text{in}} &&=\frac{1}{4}\frac{1}{1-\delta^2},\\
s^1_{(1,1)\text{out}}: &\quad \frac{1}{9}+\delta\left\{\sigma^1_{(1,1)\text{out}}\mu^1_{(0,0)}+\sigma^1_{(1,1)\text{in}}\frac{1}{4}\frac{\delta}{1-\delta^2}\right\} &&+0 &&=\mu^1_{(1,1)},\\
s^1_{(1,1)\text{in}}: &\quad \frac{1}{9}+\delta\left\{\sigma^1_{(1,1)\text{out}}\frac{1}{4}\frac{1}{1-\delta^2}+\sigma^1_{(1,1)\text{in}}\mu^1_{(1,1)}\right\} &&+0 &&=\mu^1_{(1,1)}.
\end{aligned}
$$

The rows indicated by $s^1_{(0,1)\mathrm{in}}$ and $s^1_{(1,0)\mathrm{out}}$ are trivially satisfied and therefore redundant. The rows indicated by $s^1_{(0,1)\mathrm{out}}$ and $s^1_{(1,0)\mathrm{in}}$ only have to be used for verifying that the value of the λ's appearing in these rows are nonnegative.

We consider next whether there are candidate equilibria where the probability by which a firm enters in state $(0,0)$ is equal to the probability by which it stays in state $(1,1)$: $\sigma^1_{(0,0)\mathrm{in}} = \sigma^1_{(1,1)\mathrm{in}} = \sigma$. We find from the first and the seventh (or second and eighth) equation that $\mu^1_{(0,0)} = \mu^1_{(1,1)} - \dfrac{1}{9} = \mu$. As a result, only two equations remain:

$$\delta\left\{(1-\sigma)\mu + \sigma\frac{1}{4}\frac{\delta}{1-\delta^2}\right\} = \mu,$$

$$\delta\left\{(1-\sigma)\frac{1}{4}\frac{1}{1-\delta^2} + \sigma\left(\mu + \frac{1}{9}\right)\right\} = \mu.$$

From the first equation it follows that

$$\mu = \frac{1}{4}\,\frac{\delta^2}{1-\delta^2}[1-\delta(1-\sigma)]^{-1}\sigma.$$

Substituting this in the second equation gives

$$\frac{1}{4}\,\frac{\delta}{1-\delta^2}(1-\sigma) + \frac{1}{9}\,\delta\sigma = (1-\delta\sigma)\left\{\frac{1}{4}\,\frac{\delta^2}{1-\delta^2}[1-\delta(1-\sigma)]^{-1}\sigma\right\},$$

from which the second degree equality

$$\left\{\frac{4}{9}\,\delta^3 - \delta^2 + \frac{5}{9}\,\delta\right\}\sigma^2 + \left\{-\frac{4}{9}\,\delta^3 + \frac{4}{9}\,\delta^2 - \frac{5}{9}\,\delta + \frac{5}{9}\right\}\sigma + \{-1+\delta\} = 0$$

follows. Solving this equality gives the probability of entry when no firm is present and the probability of staying active when both firms are present,

$$\sigma = \frac{-(\frac{5}{9} - \frac{5}{9}\delta + \frac{4}{9}\delta^2 - \frac{4}{9}\delta^3) + \sqrt{(\frac{5}{9} - \frac{5}{9}\delta + \frac{4}{9}\delta^2 - \frac{4}{9}\delta^3)^2 - 4(\frac{5}{9}\delta - \delta^2 + \frac{4}{9}\delta^3)(-1+\delta)}}{2(\frac{5}{9}\delta - \delta^2 + \frac{4}{9}\delta^3)}.$$

For $\delta = 0.95$, this probability is equal to

$$\sigma = \frac{-287 + \sqrt{127969}}{76} \approx 0.9306.$$

In Table 2 in the text, the data on all σ's and μ's are displayed. For instance $\sigma^2_{(0,0)\mathrm{in}}$ stands for the probability by which firm 2 enters when the current state prescribes no firms being active and is equal to 0.9306, which corresponds to the analytical solution derived above. The quantity $\mu^1_{(0,1)}$ denotes the total discounted expected profit for firm 1 when initially only firm 2 is active and is equal to 2.4359. By substituting this data in the original system of equalities and inequalities, it can be verified that we have found a Markov perfect equilibrium indeed.

For instance, substituting the data of Table 2 in the equation corresponding to the strategy $s^1_{(1,1)\mathrm{out}}$ leads to

$$0.1111 + 0.95[0.9306 \times 2.4359 + 0.0694 \times 2.3055] + \lambda^1_{(1,1)\mathrm{out}} - 2.4166 = 0$$

or, equivalently

$$\lambda^1_{(1,1)\mathrm{out}} = 2.4166 - \{0.1111 + 0.95[0.9306 \times 2.4359 + 0.0694 \times 2.3055]\}.$$

The right-hand side is the difference between the equilibrium payoff to firm 1 with initially both firms being active and the payoff when firm 1 deviates to playing strategy $s^1_{(1,1)\mathrm{out}}$ with probability one, i.e., exiting the industry for sure, in the initial stage only. Since, $\lambda^1_{(1,1)\mathrm{out}}$ equals zero, there is no gap between the two payoffs, as is typical when a mixed strategy is played. Therefore, exiting the industry with positive probability is an optimal action (given that the opponent plays according to its alternating monopoly equilibrium strategy).

When we substitute the data in the equation corresponding to the strategy $s^1_{(1,0)\mathrm{in}}$ of the same block of equations, we obtain

$$0.2500 + 0.95[1.0000 \times 2.4166 + 0.0000 \times 2.5641] + \lambda^1_{(1,0)\mathrm{in}} - 2.5641 = 0,$$

which can be rewritten as

$$\lambda^1_{(1,0)\mathrm{in}} = 2.5641 - \{0.2500 + 0.95[1.0000 \times 2.4166 + 0.0000 \times 2.5641]\}.$$

Since $\lambda^1_{(1,0)\mathrm{in}}$ equals 0.0183, this implies a positive gap between the equilibrium payoff to firm 1 when initially only firm 1 is active and the payoff when firm 1 deviates to playing strategy $s^1_{(1,0)\mathrm{in}}$ with probability one, i.e., stay active, in the initial stage. Therefore, staying active with positive probability at a state where firm 1 is a monopolist is a suboptimal action when firm 2 plays according to the alternating monopoly equilibrium.

After substituting all data in all equations and inequalities that characterizes the set of Markov perfect equilibria, it can be observed that all shadowprices except $\lambda^1_{(0,1)\mathrm{out}}$, $\lambda^1_{(1,0)\mathrm{in}}$, $\lambda^2_{(0,1)\mathrm{in}}$, and $\lambda^2_{(1,0)\mathrm{out}}$ are zero. This means that playing the actions $s^1_{(0,1)\mathrm{out}}$ and $s^1_{(1,0)\mathrm{in}}$ with positive probability has a negative effect on the total discounted profit for firm 1.

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