This six chapter monograph deals with a class of constrained optimization problems that are called Mathematical Programs with Equilibrium Constraints (MPEC). An MPEC is an optimization problem subject to constraints, some of which are of the variational inequality type. The latter constraints are therefore not given in closed form, but involve a kind of optimization problem that is related to a fixed point problem. Variational inequality problems arise naturally from equilibrium conditions as used in for instance game theory or economics. A typical example leading to an MPEC is an institution that optimizes an objective function, thereby taking into account the optimizing behaviour of agents affected by and responding to its decisions. An MPEC generalizes many classes of problems, of which the nonlinear complementarity problem is probably the best-known one.

Chapter 1 gives an impressive description of many source problems that fit within the MPEC framework. Various equivalent formulations of an MPEC are presented and some results whether optimal solutions exist are provided. To solve an MPEC it is essential to represent the variational inequality constraints as a finite system of equalities and inequalities.

Chapter 2 presents a theory of exact penalty functions for subanalytic MPEC's. In this approach (some of the) constraints are included in the objective function, where a violation of a constraint give rise to a penalty. An extensive treatment of penalty functions for many special classes of MPEC's is provided.

Chapter 3 gives a variety of constraint qualifications for MPEC under which stationary solutions can be characterized. This chapter makes clear that a straightforward application of the constraint qualifications used in classical nonlinear programming is inappropriate for MPEC.

Chapter 4 verifies the hypotheses needed for the first-order conditions of Chapter 3 for a number of special cases, for instance constraints of the affine variational inequality type, or locally unique solutions to the variational inequality constraints.

Chapter 5 contains results on necessary and sufficient second-order conditions for a local optimum of an MPEC. It discusses a multiplier-based approach, an implicit programming approach, and a piecewise programming approach to the derivation of such conditions. As in Chapter 4 special attention is given to the case where the variational inequality type constraints are affine.

Chapter 6 presents four iterative algorithms for computing a stationary point of an MPEC. The first two algorithms are based on a interior point approach involving penalties for the violation of certain constraints. These algorithms can be applied to MPEC's with monotone inner problems. The third algorithm is based on an implicit programming approach and is of the iterative descend type. The feasible region is never left and the variational inequality constraints are solved for implicitly. The fourth algorithm is based on a piecewise programming approach and is an extension to a piecewise smooth setting of Newton methods for solving smooth nonlinear programs. This algorithm is locally superlinearly, and under mild assumptions quadratically, convergent. The chapter ends by reporting some computational results with the penalty interior point approach.

An important motivation to study MPEC's is that they arise frequently in economics and game theory. For those disciplines it is crucial to have algorithms that converge to a global optimum under weak conditions. Unfortunately, the methods treated in the monograph can only be shown to converge to local optima under conditions that are often not satisfied in economic models. Nevertheless, this well-written monograph is an impressive piece of work. It provides a self-contained, state of the art treatment of the topic and points at a variety of interesting open problems. This makes the monograph worthwile to read for any one interested in the most recent developments in mathematical programming.