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Constrained suboptimality when prices are non-competitive

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ABSTRACT

The paper addresses the following question: how efficient is the market system in allocating resources if trade takes place at prices that are not competitive? Even though there are many partial answers to this question, an answer that stands comparison to the rigor by which the first and second welfare theorems are derived is lacking. We first prove a "Folk Theorem" on the generic suboptimality of equilibria at non-competitive prices. The more interesting problem is whether equilibria are constrained optimal, i.e. efficient relative to all allocations that are consistent with prices at which trade takes place. We discuss an optimality notion due to Bénassy, and argue that this notion admits no general conclusions. We then turn to the notion of p-optimality and give a necessary condition, called the separating property, for constrained optimality: each constrained household should be constrained in each constrained market. If the number of commodities is less than or equal to two, the case usually treated in the textbook, then this necessary condition is also sufficient. In that case equilibria are constrained optimal. When there are three or more commodities, two or more constrained households, and two or more constrained markets, this necessary condition is typically not sufficient and equilibria are generically constrained suboptimal.

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1. Introduction

More than two centuries ago, Adam Smith described how the pursuit of self-interest can promote the interest of society. Since then, economists have devoted much of their time in providing rigorous foundations to this claim, which finally resulted in the first and second fundamental welfare theorems. These theorems are valid only in idealized circumstances, with all trade taking place at competitive prices, including trade in contracts contingent on all imaginable future events. The case where the assumption of complete financial markets is relaxed has received much attention in the recent literature. When markets are incomplete, then a competitive equilibrium is typically suboptimal. The appropriate question to ask, however, is not whether competitive equilibria are optimal, but whether competitive allocations are optimal relative to the restrictions imposed by market incompleteness. When a fully informed central planner, who takes into account the implications of market incompleteness, is able to improve upon a competitive allocation, then competitive equilibria are said to be constrained suboptimal. Geanakoplos and Polemarchakis (1986) show that competitive equilibria are typically constrained suboptimal, by showing that Pareto improvements can be obtained by making the appropriate redistributions in households' initial asset portfolios and next restricting all trade in asset markets. More recently, similar results have been obtained that show the possibility of generating Pareto improvements by introducing new financial assets, see Cass and Citanna (1998), or Citanna

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et al. (1998) for a more general perspective, and the possibility of generating Pareto improvements by price regulation, see Polemarchakis (1979); Drèze and Gollier (1993); Drèze (2001), and Herings and Polemarchakis (2005).

The assumption that all trade takes place at competitive prices has also been relaxed. During the last quarter of the 20th century, traditional Walrasian theory has accommodated in a general equilibrium setting the possibility of sluggish price adjustment, short-run price rigidities, and, as a consequence, non-clearing markets. For seminal contributions, see Bénassy (1975); Drèze (1975), and Younès (1975). For more recent treatments, see Herings (1996) and Drèze (1997), as well as Citanna et al. (2001). Attention has been focused on issues of equilibrium existence, the possibility of coordination failures, and on explanations why prices and wages may not adjust freely to equate supply and demand. Instances of the latter are cases with information imperfectness, menu costs, renegotiation costs and so on. Many empirical studies show that quantity constraints, like involuntary unemployment in the labor market, and infrequent price adjustments, like nominal wage rigidities, are common in the real world, see Romer (1996). Other examples where the analysis of non-clearing markets is relevant are situations involving market power, planned economies and markets for agricultural products, see Bénassy (1993). More generally, application of standard tools from public choice theory shows that governments have incentives to intervene in the price formation process to gain votes, see Herings (1997) and Tuinstra (2000).

Most economists share the strong conviction that imperfections in the price formation process, and in particular trade at non-competitive prices, has strongly negative welfare consequences. Given the strength of this conviction, it is surprising that most of its foundations come from partial settings. No rigorous general results that stand comparison to the first and second welfare theorems, or the constrained suboptimality results in the case of market incompleteness, are available. It is therefore that we label the claim on the detrimental effects of trade at non-competitive prices as a Folk Theorem.

The paper addresses the following question: how efficient is the market system in allocating resources when prices are not competitive. To get an answer, we analyze the equilibria of the cleanest fixed price model available, the one of Drèze (1975). In his seminal paper, Drèze introduced the concept of quantity rationing in a general equilibrium model with price rigidities. In this approach a household chooses that commodity bundle which is most preferred by it, subject to both the budget constraint and the quantity constraints on net trades. The quantity rationing may affect either supply or demand of a commodity, but it never affects both simultaneously to reflect the transparency of markets. The first main result we show is the Folk Theorem on the generically suboptimal allocation of resources when prices are non-competitive.

Inspired by the incomplete markets literature, we continue our investigation by analyzing a concept of constrained optimality, that takes into account the restrictions imposed by trading at false prices. Two constrained optimality notions have been provided in the literature, one by Bénassy (1975) and one by Younès (1975). We refer to the former one as B-p-optimality and the latter one as p-optimality. The notion of B-p-optimality involves a chain of pairs of households retrading pairs of commodities. The notion is satisfied if such a chain does not exist. An allocation is p-optimal if it is efficient relative to the set of physically feasible allocations for which the net trades of all households have value zero at the price vector p. We show that p-optimality strengthens B-p-optimality. We provide robust examples showing that no general conclusions regarding B-p-optimality of a Drèze equilibrium can be drawn.

Böhm and Müller (1977) give an example of an economy, whose equilibria are not p-optimal. Maskin and Tirole (1984) observe that if all traders have strictly positive weights in a welfare program, then non-competitive p-optimal allocations involving trade in all markets are never voluntary, that is imply forced trade, and, therefore, are not equilibria. However, satiation or non-constrained maximization is typical for a fixed price model, see Aumann and Drèze (1986), which implies that p-optimal allocations need not be solutions to welfare programs with strictly positive weights. The question rises whether trade at non-competitive prices leads typically to constrained suboptimal allocations.

A household is said to be constrained if it is subject to quantity rationing in at least one market. A market is said to be constrained if at least one household faces constraints in that market. We give an easy to verify necessary condition for equilibria to be p-optimal: each constrained household should be constrained in each constrained market. If the number of commodities is less than or equal to two, then this necessary condition is also sufficient. In that case equilibria are constrained optimal. This case corresponds to the general equilibrium equivalent of the partial equilibrium textbook picture that analyzes the effects of a minimum or a maximum price on a single good. This case is misleading. In cases with more than two goods, and at least two constrained households, this necessary condition is not sufficient and generic constrained suboptimality of equilibria is obtained.

Our results remain valid when most markets are characterized by full price flexibility and absence of any form of rationing. We show that our results carry over to the set-up of Citanna et al. (2001), where price rigidities are rationing prevail only on some markets, and all other markets are cleared by price adjustments.

The paper has been organized as follows. Section 2 exposes a model of an exchange economy where trade takes place at non-competitive prices. Section 3 shows that in such an economy, equilibria are typically suboptimal. Section 4 introduces B-p-optimality and p-optimality and provides robust examples showing that the former notion may or may not hold at equilibrium. Section 5 turns to the stronger version of p-optimality and shows that in the two commodity case constrained optimality holds. Section 6 derives the necessary condition that all constrained households be constrained in each constrained market for constrained optimality to hold. It is shown that this criterion is typically not met when the number of commodities is greater than or equal to three. Section 7 extends the analysis to the case where full price flexibility prevails on most markets. Finally, Section 8 concludes.

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2. The model

We consider an exchange economy denoted by $\mathcal{E} = < \mathcal{N}, \mathcal{L}, \{X^i, u^i, w^i\}_{i \in \mathcal{N}} >$. Here $\mathcal{N} = \{1, \ldots, N\}$ is the set of households, indexed by i, and $\mathcal{L} = \{0, 1, \ldots, L\}$ is the set of commodities, indexed by l. Each household is characterized by a consumption set X^i, a subset of $\mathbb{R}^{L+1}$, a utility function u^i defined on X^i, and a vector of initial endowments w^i in X^i. Price systems belong to the set $P = \{p \in \mathbb{R}^{L+1} \,|\, p_0 = 1,\ p \gg 0\}$. Commodity 0 serves as a numeraire commodity.

To evaluate the welfare consequences of trade at non-competitive prices, we consider a, not necessarily competitive, price system p in P at which trade actually takes place. In general, since prices p might be not compatible with a competitive equilibrium, traders will face quantity constraints on supply and demand. The description of the market mechanism is now extended in the sense that the information transmitted by it is no longer only the price system, but also the maximal amount a household is able to supply of every commodity, called the rationing scheme on supply, and the maximal amount a household is able to demand of every commodity, called the rationing scheme on demand. In this we follow the approach and formulation of Drèze (1975). All exchange takes place against the numeraire commodity, which is not rationed. A household faces rationing $\underline{z}_l \in \mathbb{R}_-$ and $\bar{z}_l \in \mathbb{R}_+$ in the market for each commodity $l \neq 0$, which represents the minimal and the maximal amount of good l household i is able to trade. For the sake of simplicity we only consider uniform rationing, which means that $\underline{z}_l$ and $\bar{z}_l$ are the same for every household i.

Given a price system p and a rationing scheme $(\underline{z}, \bar{z}) \in \mathbb{R}^L_- \times \mathbb{R}^L_+$, a household maximizes its utility function u^i over its constrained budget set defined by

$$B^i(\underline{z}, \bar{z}, p) = \{x^i \in X^i \,|\, px^i \leq pw^i \quad \text{and} \quad \underline{z}_l \leq x^i_l - w^i_l \leq \bar{z}_l, \quad l = 1, \ldots, L\}.$$

Throughout the paper we will use the following assumptions with respect to the economy $\mathcal{E}$:

A1 For every household $i \in \mathcal{N}$, $X^i = \mathbb{R}^{L+1}_{++}$.

A2 For every household $i \in \mathcal{N}$, the utility function u^i is C^2 on X^i, u^i is differentiably strictly increasing, i.e., $Du^i(x^i) \gg 0$ for all $x^i \in X^i$, and u^i is differentiably strictly quasiconcave, i.e. the Gaussian curvature of $I_{x^i} = \{y^i \in X^i | u^i(y^i) = u^i(x^i)\}$ is different from zero for any $x^i \in X^i$. Moreover, u^i satisfies the boundary condition, that is, for every $x^i \in X^i$ the set $\{y^i \in X^i \mid u^i(y^i) \geq u^i(x^i)\}$ is closed relative to $\mathbb{R}^{L+1}$.

A3 For every household $i \in \mathcal{N}$, $w^i \in \mathbb{R}^{L+1}_{++}$.

The demand function of individual i is defined by

$$d^i(\underline{z}, \bar{z}, p) = \operatorname{argmax}_{x^i \in B^i(\underline{z}, \bar{z}, p)} u^i(x^i).$$

Assumptions A1–A3 guarantee that d^i is a function indeed. Given $(\underline{z}, \bar{z}, p) \in \mathbb{R}^L_- \times \mathbb{R}^L_+ \times P$, household i is said to be constrained in its supply in market l if for any $\underline{\tilde{z}} \in \mathbb{R}^L_-$, such that $\underline{\tilde{z}}_k = \underline{z}_k$ for $k \neq l$ and $\underline{\tilde{z}}_l = \underline{z}_l - \varepsilon$ for some positive ε, $u^i(d^i(\underline{\tilde{z}}, \bar{z}, p)) > u^i(d^i(\underline{z}, \bar{z}, p))$. If household i is constrained in its supply in the market for commodity l, then it follows from the strict quasi-concavity of the utility function that $d^i_l(\underline{z}, \bar{z}, p) - w^i_l = \underline{z}_l$. A household that is constrained in its supply in market l can improve its utility if the rationing on supply in market l is relaxed. A similar definition is made with respect to rationing on demand. The market for commodity l is said to be constrained if there is at least one household constrained in it, either in supply or in demand.

Following Drèze (1975), we introduce a Drèze equilibrium of the economy $\mathcal{E}$ at prices $p \in P$.

Definition 2.1. A *Drèze equilibrium* at prices $p \in P$ of an economy $\mathcal{E}$ is an allocation $(\bar{x}^1, \ldots, \bar{x}^N) \in \prod_{i \in \mathcal{N}} X^i$ such that there exists $(\underline{z}, \bar{z}) \in \mathbb{R}^L_- \times \mathbb{R}^L_+$ satisfying the following conditions:

(i) for all $i \in \mathcal{N}$, $\bar{x}^i$ maximizes u^i on $B^i(\underline{z}, \bar{z}, p)$;
(ii) $\sum_{i \in \mathcal{N}} \bar{x}^i = \sum_{i \in \mathcal{N}} w^i$;
(iii) for every $l \in \mathcal{L} \setminus \{0\}$,

$$\bar{x}^{i'}_l - w^{i'}_l = \bar{z}_l \ \text{for some}\ i' \in \mathcal{N} \ \text{implies}\ \bar{x}^i_l - w^i_l > \underline{z}_l \quad \text{for all}\ i \in \mathcal{N},$$

$$\bar{x}^{i'}_l - w^{i'}_l = \underline{z}_l \ \text{for some}\ i' \in \mathcal{N} \ \text{implies}\ \bar{x}^i_l - w^i_l < \bar{z}_l \quad \text{for all}\ i \in \mathcal{N}.$$

The first two conditions of the definition are standard. They state that every household behaves optimally given the price system and the rationing scheme, and that all markets clear. Condition (iii) guarantees that markets are transparent. Constraints are on one side of the market at most. The requirement that $(\underline{z}, \bar{z})$ belong to $\mathbb{R}^L_- \times \mathbb{R}^L_+$ implies that there is no forced trading. Nothing precludes that the prices p are competitive. A competitive equilibrium is indeed a special case of a Drèze equilibrium. It is a Drèze equilibrium without binding rationing.

Notice that the case with two commodities, $L = 1$, is the exact general equilibrium analogue of the standard textbook analysis of partial equilibrium, where, for instance, a minimum price is imposed in the market for commodity 1, which is

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exchanged against commodity 0. If at the minimum price supply exceeds demand, which is the case with standard upward sloping supply curves and downward sloping demand curves, then the quantity actually traded is determined by the short side of the market, the total demand for commodity 1. Some of the suppliers will be constrained. They are only able to supply part of their preferred supply.

Now consider the general equilibrium set-up with $L = 1$ and suppose that at prices p total net supply exceeds total net demand. A Drèze equilibrium will necessarily involve only rationing on the supply side, so $\bar{z}_1$ equals a number sufficiently large not to affect the households' decision problems. Total net demand is not affected by rationing on the supply side, so when $\underline{z}_1$ is such that constrained net supply equals total net demand for commodity 1, the unique Drèze equilibrium is obtained.

Closely related to the Drèze equilibrium are the equilibrium concepts of Bénassy (1975) and Younès (1975). In Drèze's model the demand expressed by a household satisfies the constraints imposed by the rationing schemes. In Bénassy (1975) a household expresses his effective demand. In his effective demand for a commodity, the household takes into account the rationing schemes on all markets, except on the market of that commodity. As is shown in Grandmont (1977), the equilibrium concept given in Bénassy (1975) might cause inconsistencies when the preference relations of the households are not strictly convex, in the sense that the final consumption bundle obtained by a household is not feasible for him, and even if it is feasible, it might not be optimal given the constraints the household perceives. The equilibrium concepts related to the Drèze equilibrium are frequently used in the macro-economic disequilibrium models, see Malinvaud (1977), Bénassy (1982), and Böhm (1989). In Silvestre (1982) conditions are given under which the approaches of Bénassy, Drèze, and Younès are equivalent. These conditions are implies by A1–A3, so the choice of equilibrium concept does not affect our results.

3. Suboptimality of equilibrium

Our first aim is to prove the Folk Theorem that, given a tuple of utility functions $u^1, \ldots, u^N$, and prices $p \in P$, for almost all initial endowments $(w^1, \ldots, w^N) \in \mathbb{R}^{N(L+1)}_{++}$ every Drèze equilibrium is suboptimal. An allocation $(x^1, \ldots, x^N)$ is said to be feasible if $x^i \in X^i$, $i \in \mathcal{N}$, and $\sum_{i \in \mathcal{N}} x^i = \sum_{i \in \mathcal{N}} w^i$.

Definition 3.1. A feasible allocation $(x^1, \ldots, x^N)$ is *optimal* if there is no feasible allocation y such that $u^i(y^i) \geq u^i(x^i)$ for all $i \in \mathcal{N}$ with at least one inequality strict.

As a consequence of the boundary condition on the utility function, an allocation $(x^1, \ldots, x^N)$ is optimal if and only if

$$\frac{\partial_{x^i} u^i(x^i)}{\partial_{x^i_0} u^i(x^i)} = \frac{\partial_{x^{i'}} u^{i'}(x^{i'})}{\partial_{x^{i'}_0} u^{i'}(x^{i'})}$$

for every $i, i' \in \mathcal{N}$.

Lemma 3.2. *Suppose that $(\bar{x}^1, \ldots, \bar{x}^N)$ is a Drèze equilibrium at prices $p \in P$ of an economy $\mathcal{E}$ for a rationing scheme $(\underline{z}, \bar{z}) \in \mathbb{R}^L_- \times \mathbb{R}^L_+$. Then there exist $\lambda^i \in \mathbb{R}_{++}$, $\underline{\mu}^i_l, \bar{\mu}^i_l \in \mathbb{R}_+$, $i \in \mathcal{N}$, $l \in \mathcal{L} \setminus \{0\}$, such that*

$$\frac{\partial_{x^i_l} u^i(\bar{x}^i)}{\partial_{x^i_0} u^i(\bar{x}^i)} = p_l - \frac{\underline{\mu}^i_l - \bar{\mu}^i_l}{\lambda^i}, \quad i \in \mathcal{N}, l \in \mathcal{L} \setminus \{0\},$$

$\lambda^i = \partial_{x^i_0} u^i(\bar{x}^i)$, *$\underline{\mu}^i_l > 0$ implies $x^i_l - w^i_l = \underline{z}_l$, and $\bar{\mu}^i_l > 0$ implies $x^i_l - w^i_l = \bar{z}_l$.*

Proof. The conclusion of the lemma follows from the first-order conditions for the maximization problem of household i:

$$\begin{array}{ll} \max_{x^i \in X^i} & u^i(x^i) \\ \text{s.t.} & px^i - pw^i \leq 0, \\ & \underline{z}_l \leq x^i_l - w^i_l \leq \bar{z}_l, \; l \in \mathcal{L} \setminus \{0\}. \end{array}$$

The Lagrangian of the problem is

$$\mathcal{L}^i(x^i, \lambda^i, \underline{\mu}^i, \bar{\mu}^i) = u^i(x^i) - \lambda^i(px^i - pw^i) - \sum_{l=1}^{L} \left(\underline{\mu}^i_l(\underline{z}_l - x^i_l + w^i_l) + \bar{\mu}^i_l(-\bar{z}_l + x^i_l - w^i_l) \right).$$

The derivatives of $\mathcal{L}^i$ with respect to x^i are equal to zero at $\bar{x}^i$:

$$\partial_{x^i_0} \mathcal{L}^i = \partial_{x^i_0} u^i(\bar{x}^i) - \lambda^i = 0,$$
$$\partial_{x^i_l} \mathcal{L}^i = \partial_{x^i_l} u^i(\bar{x}^i) - \lambda^i p_l + \underline{\mu}^i_l - \bar{\mu}^i_l = 0, \quad l \neq 0.$$

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Notice that λ^i is never equal to 0, since the numeraire commodity is always desirable. The first part of Lemma 3.2 is now straightforward. Moreover, the Kuhn-Tucker conditions imply that

$$\underline{\mu}_l^i(\underline{z}_l^i - x_l^i + w_l^i) = 0,$$

$$\bar{\mu}_l^i(\bar{z}_l^i - x_l^i + w_l^i) = 0,$$

which gives the second part of the lemma. □

The interpretation of the lemma is very natural. The marginal rate of substitution between good l and the numeraire equals the price of good l, if the household is unconstrained in market l. It is less than p_l, if the household is constrained in its supply in market l, and it is greater than p_l, if the household is constrained in its demand in market l. Drèze equilibria may be optimal, for instance, if the initial allocation of resources is optimal. This is necessarily the case if there is just one commodity or just one household. But when the number of commodities and households is greater than one, we show this situation to be rather exceptional.

When $L \geq 2$ and $N \geq 2$, we can construct an example of an economy with an optimal Drèze equilibrium at non-competitive prices $\bar{p}$ but an inefficient initial distribution of resources. Suppose that there are two households and three commodities, and competitive equilibrium prices p^* are such that the excess demand for commodity 2 is zero for both households, so only two goods are traded in non-zero amounts. Consider prices $\bar{p}$ such that $\bar{p}_2 < p_2^*$, and $\bar{p}_l = p_l^*$, $l = 0, 1$. It is possible to choose utility functions such that the demand for good 2 becomes positive for both households under the price system $\bar{p}$. In fact, this is the natural case. Choose a rationing scheme $(\underline{z}, \bar{z})$ with $\bar{z}_2 = 0$ and other components of $(\underline{z}, \bar{z})$ non-binding. Strict quasi-concavity of preferences implies that $\bar{z}_2 = 0$ is binding in an optimal solution to the household's decision problem. But then the competitive allocation is a Drèze equilibrium of the economy $\mathcal{E}$ at prices $\bar{p}$, which means that the Drèze equilibrium is optimal.

For $L = 1$ such an example cannot be constructed. If $L = 1$ and the initial distribution of resources is inefficient, then a Drèze equilibrium at non-competitive prices p is necessarily inefficient. Suppose $L = 1$ and let $\bar{x}$ be an optimal Drèze equilibrium at non-competitive prices p of an economy $\mathcal{E}$, whereas the initial resources of $\mathcal{E}$ are distributed inefficiently. There is at least one household i who is rationed, and at least one household i', who is not. The former holds because p is non-competitive. The latter because trade is needed to go from an inefficient initial distribution of resources to an optimal allocation. Condition (iii) of Definition 2.1 implies that one side of the market of commodity 1 is unconstrained. By Lemma 3.2 it holds that

$$\frac{\partial_{x^i} u^i(\bar{x}^i)}{\partial_{x_0^i} u^i(\bar{x}^i)} \neq \frac{\partial_{x^{i'}} u^{i'}(\bar{x}^{i'})}{\partial_{x_0^{i'}} u^{i'}(\bar{x}^{i'})},$$

which contradicts the optimality of $\bar{x}$.

The following proposition gives a useful characterization of optimal Drèze equilibria. It claims that each optimal Drèze equilibrium coincides with a competitive equilibrium allocation. This characterization is needed to show the generic suboptimality of Drèze equilibria.

Proposition 3.3. *A Drèze equilibrium $\bar{x}$ at prices p of an economy $\mathcal{E}$ for a rationing scheme $(\underline{z}, \bar{z})$ is optimal if and only if $\bar{x}$ corresponds to a competitive equilibrium allocation.*

Proof. Let $\bar{x}$ be an optimal Drèze equilibrium at prices p and a rationing scheme $(\underline{z}, \bar{z})$. Optimality together with Assumption A1 implies that

$$\frac{\partial_{x_l^i} u^i(\bar{x}^i)}{\partial_{x_0^i} u^i(\bar{x}^i)} = \frac{\partial_{x_l^{i'}} u^{i'}(\bar{x}^{i'})}{\partial_{x_0^{i'}} u^{i'}(\bar{x}^{i'})}, \quad \text{for any } i, i' \in \mathcal{N}.$$

Then it follows from Lemma 3.2 that $(\underline{\mu}_l^i - \bar{\mu}_l^i)/\lambda^i$ does not depend on i. Together with Condition (iii) in the definition of a Drèze equilibrium, it implies that for each market $l \in \mathcal{L} \setminus \{0\}$, either every $\underline{\mu}_l^i$ is positive, or every $\bar{\mu}_l^i$ is positive, or both these multipliers are equal to zero. The first two cases mean that everyone in market l is constrained, so from market clearing it follows that $\bar{x}_l^i = w_l^i$, for all $i \in \mathcal{N}$. The last possibility is equivalent to the situation of a free market without rationing. Consider a vector of prices $p' \in P$ such that

$$p_0' = p_0 = 1,$$
$$p_l' = p_l - \frac{\underline{\mu}_l^i - \bar{\mu}_l^i}{\lambda^i}, \quad l \neq 0.$$

Since prices p' are different from p only for markets without trade, $\bar{x}$ satisfies the budget condition under the price system p', i.e. $p'\bar{x}^i = p'w^i$, $i \in \mathcal{N}$. It follows immediately that $\bar{x}$ is a competitive equilibrium allocation at competitive prices p', which proves the "only if" part of the proposition.

It is immediate that a Drèze equilibrium $\bar{x}$, which corresponds to a competitive equilibrium allocation, is optimal. □

The next step is to show the generic suboptimality of Drèze equilibria.

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Theorem 3.4. *Fix any price system $\bar{p} \in P$ and utility functions $u^1, \ldots, u^N$ satisfying Assumption A2. Then there is an open set of full Lebesgue measure of initial endowments in $\mathbb{R}^{N(L+1)}_{++}$ such that every Drèze equilibrium at prices $\bar{p}$ of the economy $\mathcal{E}$ is suboptimal.*

Proof. By Proposition 3.3, an optimal Drèze equilibrium corresponds to a competitive equilibrium allocation. It follows from the results in Laroque (1978) that for an open set of full Lebesgue measure of initial endowments, for every competitive equilibrium allocation x^*:

$$x_l^{*i} - w_l^i \neq 0,$$

for every household i and every commodity l. Therefore, using Lemma 3.2, generically in initial endowments, cases where all $\underline{\mu}_l^i > 0$ and all $\bar{\mu}_l^i > 0$ are excluded. Generically in endowments, an optimal Drèze equilibrium consists of competitive equilibrium prices and a competitive equilibrium allocation.

To complete the proof we need to show that for generic w the price system $\bar{p}$ is not competitive. Let $z(p, w)$ denote aggregate excess demand at prices p and endowments $w = (w^1, \ldots, w^N)$. Let $F_l(p, w)$ be equal to $z_l(p, w)$ for $l = 1, \ldots, L$, and define $F_{L+1}(p, w) = p_L - \bar{p}_L$. If (p, w) is such that $z(p, w) = 0$, then the rank of the matrix $\partial_w z(p, w)$ is L, see Mas-Colell (1985), p. 227. Therefore, the rank of the system $\partial_{w,p_L} F(p, w)$:

$\partial_w z(p, w)$	$\partial_{p_L} z(p, w)$
0	1

is equal to $L + 1$ if $F(p, w) = 0$. By the Transversality Theorem, $\partial_{\tilde{p}} F^w(p)$ has full rank for almost all $w \in \mathbb{R}^{N(L+1)}_{++}$ if $F^w(p) = 0$, where $F^w(p) = F(p, w)$, $\tilde{p} = (p_1, \ldots, p_L)$. Since the rank of $\partial_{\tilde{p}} F^w(p)$ can be at most L Since the rank of $\partial_{\tilde{p}} F^w(p)$ can at most be L, generically in initial endowments, $z(p, w) = 0$ and $p_L = \bar{p}_L$ has no solution. Consequently, generically in initial endowments, the price system $\bar{p}$ is not competitive. We conclude that for a set of endowments of full Lebesgue measure, all Drèze equilibria are suboptimal.

We now show that we can choose the set of initial endowments of full Lebesgue measure for which all Drèze equilibria are suboptimal to be open. Notice that the set of $(p, w) \in P \times \mathbb{R}^{N(L+1)}_{++}$ such that $F(p, w) = 0$ is closed due to the continuity of F. It follows from Balasko (1988), p. 89, that the natural projection function, which maps (p, w) into w, is proper. This implies that the set of initial endowments, for which some competitive equilibrium price has its last component equal to $\bar{p}_L$, is closed, since the image of a closed set under a proper function is closed. The complement of this set is open and of full Lebesgue measure. The intersection of this set with the open set of full Lebesgue measure for which there is trade for every household for every commodity at a competitive equilibrium, is open and of full measure and contains only endowments with suboptimal Drèze equilibria. □

The theorem gives a rigorous statement of the Folk Theorem on the generic suboptimality of equilibria at non-competitive prices. The next step is whether we can even strengthen this conclusion to generic constrained suboptimality.

4. Two optimality notions

It is apparent that as long as prices are not competitive, full optimality is too much to be expected. The appropriate criterion in this case is one of constrained optimality. Two alternative notions have been introduced in the literature, one in Bénassy (1975) and an alternative one in Younès (1975). We refer to Bénassy's notion as B-p-optimality, and to the one of Younès as p-optimality.

Following Bénassy, consider an economy $\mathcal{E}$, price $p \in P$, and an agent i with a consumption plan x^i. Define a binary relation $\mathcal{R}_i$ on the set of goods by

$$l(\mathcal{R}_i)l' \Leftrightarrow \begin{cases} \partial_{x_l^i} u^i / \partial_{x_{l'}^i} u^i > p_l / p_{l'} \\ x_{l'}^i > 0. \end{cases}$$

A Pareto-improving chain is a set of goods $l_1, \ldots, l_k$ and traders $i_1, \ldots, i_k$ such that

$$l_1(\mathcal{R}_{i_1})l_2, l_2(\mathcal{R}_{i_2})l_3, \ldots, l_k(\mathcal{R}_{i_k})l_1.$$

Definition 4.1. Fix a price system $p \in P$. A feasible allocation $(x^1, \ldots, x^N) \in \prod_{i \in \mathcal{N}} X^i$ is B-p-optimal if no Pareto-improving chain exists at prices p.

The notion introduced in Younès (1975), p-optimality, requires optimality relative to all reallocations for which trades of all households have zero value at an admissible price system p.

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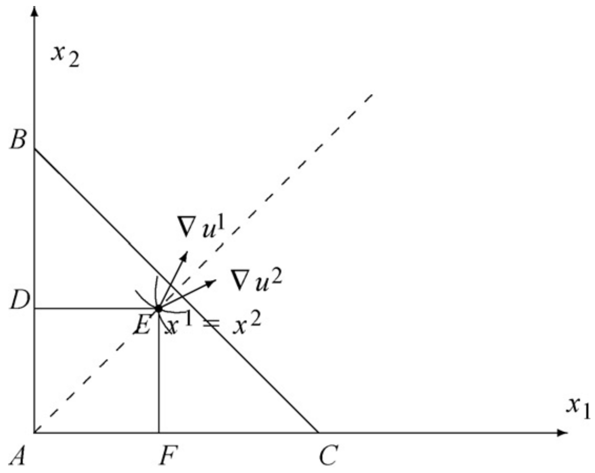

Fig. 1. B-p-suboptimal Drèze equilibrium.

Definition 4.2. Fix a price system $p \in P$. A feasible allocation $(x^1, \ldots, x^N) \in \prod_{i \in \mathcal{N}} X^i$ is p-optimal if there is no allocation $(y^1, \ldots, y^N) \in \prod_{i \in \mathcal{N}} X^i$ such that

(i) $\forall i \in \mathcal{N} \quad py^i = px^i$,
(ii) $\sum_{i \in \mathcal{N}} y^i = \sum_{i \in \mathcal{N}} w^i$,
(iii) $\forall i \in \mathcal{N} u^i(y^i) \geq u^i(x^i)$ with strict inequality for at least one $i' \in \mathcal{N}$.

Notice first that p-optimality is stronger then B-p-optimality.

Proposition 4.3. *If a feasible allocation x of an economy $\mathcal{E}$ is p-optimal, then it is also B-p-optimal.*

Proof. Since all agents have positive quantities of each good, an allocation is B-p-optimal if and only if for all commodity pairs (l, l') the numbers:

$$\frac{\partial_{x^i_l} u^I}{\partial_{x^i_{l'}} u^I} - \frac{p_l}{p_{l'}}$$

have the same sign for all households i. Let x be a p-optimal allocation and assume that it is not B-p-optimal. Then there should exist a commodity pair (l, l') and a pair of traders (i, i') such that

$$\frac{\partial_{x^i_l} u^I}{\partial_{x^i_{l'}} u^I} > \frac{p_l}{p_{l'}} > \frac{\partial_{x^{i'}_l} u^I}{\partial_{x^{i'}_{l'}} u^I}.$$

Let v^i and $v^{i'}$ be the projections of the gradient vectors $\partial_{x^i} u^i$ and $\partial_{x^{i'}} u^{i'}$ on the subspace $\{y \in \mathbb{R}^{L+1} \mid py = 0\}$. Since x is p-optimal, it should hold that $v^i = \gamma v^{i'}$ for some non-negative constant γ. Moreover, $\partial_{x^i} u^i = v^i + \alpha p$ and $\partial_{x^{i'}} u^{i'} = v^{i'} + \beta p$ for some strictly positive α and β. Then

$$v^i_l > \frac{p_l}{p_{l'}} v^i_{l'} \quad \text{and} \quad \frac{p_l}{p_{l'}} v^{i'}_{l'} > v^i_l,$$

which contradicts $v^i = \gamma v^{i'}$, $\gamma \geq 0$ and p-optimality of x. □

It is not difficult to find robust examples of B-p-optimal and B-p-suboptimal Drèze equilibria. Fig. 1 considers the following example of an economy with three goods and three households, two of whom, namely, households 1 and 2, are constrained in their demand for commodities x_1 and x_2. The triangle ABC and rectangle $ADEF$ are, respectively, the projections of households 1 and 2's Walrasian budget sets and constrained budget sets on the coordinate plane $0x_1x_2$. The prices p are equal to $(1, 1, 1)$ and ∇u^i, $i = 1, 2$, denotes the projections of the gradients of agents' utility functions on $0x_1x_2$. Fig. 1 shows a Drèze equilibrium, which is not B-p-optimal. The first household reports to a "trading post" that he would like to exchange commodity 1 for commodity 2 and the second household reports that he would like to exchange commodity 2 for commodity 1, thus there is a Pareto-improving chain

$$2(\mathcal{R}_1)1(\mathcal{R}_2)2.$$

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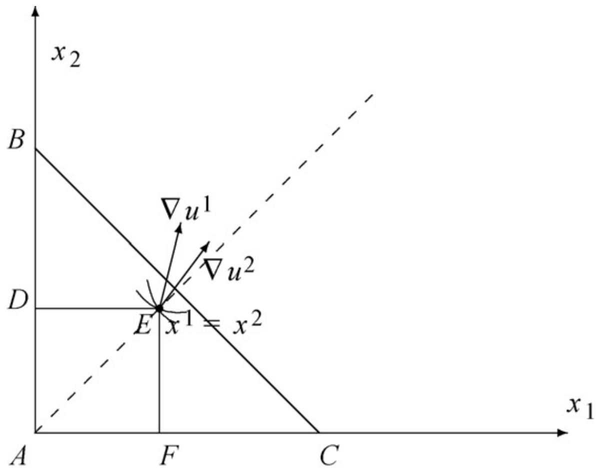

Fig. 2. B-p-optimal Drèze equilibrium.

Fig. 2 shows a Drèze equilibrium, which is B-p-optimal but not p-optimal. Both households report that they would like to exchange commodity 1 for commodity 2, so no Pareto-improving chain exists. However, there still may exist a Pareto-improving trade consistent with prices p.[1]

Our examples show that no general conclusions regarding B-p-optimality of Drèze equilibria can be drawn. The next two sections turn to p-optimality. We regard this also as the more relevant concept, as it is not clear why Pareto improving reallocations should only involve chains of pairs of commodities as opposed to reallocations of bundles of goods.

5. Constrained optimality when $L = 1$

We start by analyzing p-optimality for the case with two commodities, so $L = 1$. Strict quasi-concavity of utility functions implies that the preferences of households over the set of all attainable amounts of good 1 given fixed prices and the budget constraint are single-peaked. In this case it is possible to show uniqueness and constrained optimality of a Drèze equilibrium.

Proposition 5.1. *If $L = 1$, then a Drèze equilibrium $(\bar{x}^1, \ldots, \bar{x}^N)$ at prices $p \in P$ of an economy $\mathcal{E}$ is unique and p-optimal.*

Proof. If $L = 1$, then quantity constraints are present only on the market of commodity 1. As far as an analysis of equilibrium is concerned, there is no loss of generality by indexing all relevant rationing schemes as a function of $\underline{q}, \bar{q} \in [0, 1]$:

$$\underline{z}_1(\underline{q}) = -\underline{q}\sum_{i \in \mathcal{N}} w_1^i,$$

$$\bar{z}_1(\bar{q}) = \bar{q}\sum_{i \in \mathcal{N}} w_1^i.$$

The aggregate excess demand for commodity 1 at prices p and rationing parameters $\underline{q}, \bar{q}$ is given by

$$z_1(\underline{q}, \bar{q}) = \sum_{i \in \mathcal{N}} d_1^i(\underline{z}_1(\underline{q}), \bar{z}_1(\bar{q}), p) - \sum_{i \in \mathcal{N}} w_1^i.$$

Since households face constraints either on demand, or on supply, but not on both of them, it is possible to represent all relevant rationing schemes by a single parameter $q \in [0, 1]$ as follows:

$$\tilde{z}_1(q) = z_1(\min\{2q, 1\}, \min\{2 - 2q, 1\}).$$

Here $q = 0$ corresponds to full rationing on supply, $q = 1$ corresponds to full rationing on demand, and when $q = 1/2$ there is no rationing at all. It is immediate that $\tilde{z}_1(0) \geq 0$ and $\tilde{z}_1(1) \leq 0$. The function $\tilde{z}_1$ is continuous and is easily shown to be weakly decreasing in q.

Proposition 5.1 is clearly true if p is competitive. If p is non-competitive, assume that $\tilde{z}_1(1/2) < 0$. Let q_s be the minimal value of q such that $\tilde{z}_1(q_s) = \tilde{z}_1(1/2)$. Since the rationing scheme $\underline{z}_1(2q)$ is binding for at least one household for all $q \in [0, q_s)$, it is easy to see that the function $\tilde{z}_1$ is strictly decreasing on the interval $[0, q_s]$. Moreover, $\tilde{z}_1(0) \geq 0$ and $\tilde{z}_1(q_s) = \tilde{z}_1(1/2) < 0$.

[1] Since vectors $\nabla u^1 - \tilde{p}\partial_{x_0^1} u^1$ and $\nabla u^2 - \tilde{p}\partial_{x_0^2} u^2$, where $\tilde{p} = (p_1, p_2)$, do not have to be collinear. See Section 6 for necessary and sufficient conditions of p-optimality.

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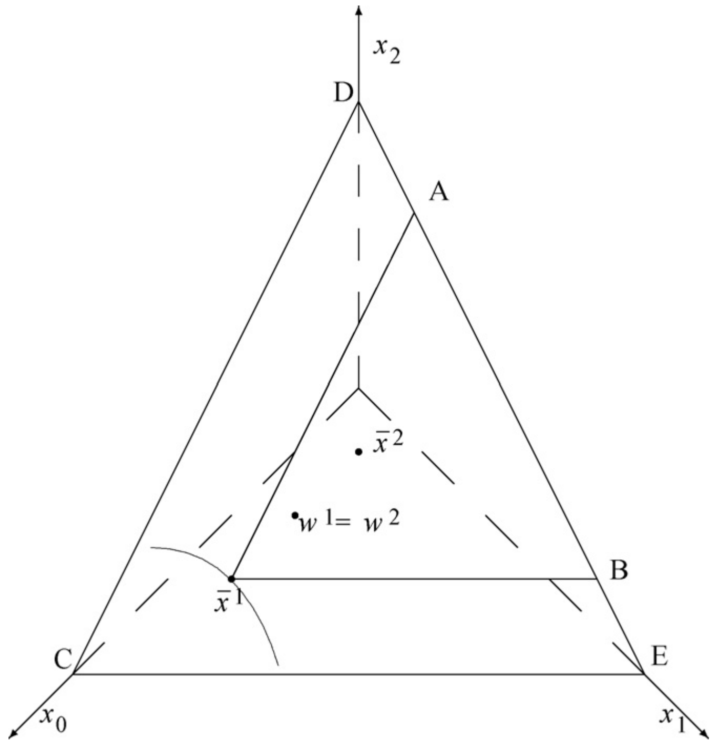

Fig. 3. Example of a p-optimal equilibrium.

This implies that a Drèze equilibrium exists and is unique. Households on the short side of the market, the demand side in this case, are not rationed and get the most preferred consumption bundle they can reach under the given fixed prices, households on the long side cannot improve without making some other household worse off. Therefore, a Drèze equilibrium is p-optimal.

A similar argument applies when $\tilde{z}_1(1/2) > 0$. □

Without any doubt the case $L = 1$ is special. We think it has some importance, as it is the case that is typically analyzed in textbooks. In Section 3 we have argued that optimality of equilibrium typically fails when $L = 1$. More precisely, we have argued before Proposition 3.3 that an equilibrium at non-competitive prices p is efficient for $L = 1$ if and only if the initial distribution of resources is efficient. Proposition 5.2 makes clear that a weaker notion of optimality, p-optimality does hold for all equilibria.

6. Constrained suboptimality when $L \geq 2$

If L is greater than or equal to 2, the situation becomes different. Then a Drèze equilibrium is not necessarily p-optimal. Two counter-examples can be found in Böhm and Müller (1977). Using a modified Edgeworth box diagram, they showed that equilibria and constrained optima constitute two disjoint sets. Robust examples of constrained optimal Drèze equilibria can be easily found as well. Fig. 3 shows such an example. There are three goods in an economy and two households endowed with the same amount of initial resources. The triangle CDE corresponds to the set $\{x^i \in X^i | px^i = pw^i\}$, where the price system p is fixed. The second household consumes its most preferred consumption bundle on CDE. The triangle $\bar{x}^1$ AB corresponds to the constrained budget set of the first household. This household faces lower bounds on the net trade in the market for both commodities 1 and 2. An indifference curve through $\bar{x}^1$ is depicted. It is easily verified that $(\bar{x}^1, \bar{x}^2)$ is a p-optimal Drèze equilibrium. Our next aim is to provide mild conditions that rule out examples like this one.

Consider a price system $p \in P$. To study the matter of constrained optimality, consider a "transformed" economy $\tilde{\mathcal{E}}$ with the same set of traders $\mathcal{N}$ as in the original economy $\mathcal{E}$, and the set of goods $\tilde{\mathcal{L}} = \mathcal{L} \setminus \{0\}$. The economy $\tilde{\mathcal{E}}$ is derived from $\mathcal{E}$ by using the budget equality to eliminate commodity 0. Initial endowments, consumption sets and utility functions of household $i \in \mathcal{N}$ are specified as follows:

$$\tilde{w}^i = w^i_{-0},$$
$$\tilde{X}^i = \{(\tilde{x}^i_1, \ldots, \tilde{x}^i_L) \in \mathbb{R}^L_{++} | \tilde{p}(\tilde{x}^i - \tilde{w}^i) \leq w^i_0\},$$
$$\tilde{u}^i(\tilde{x}^i) = u^i(w^i_0 - \tilde{p}(\tilde{x}^i - \tilde{w}^i), \tilde{x}^i_1, \ldots, \tilde{x}^i_L),$$

where $\tilde{p} = (p_1, \ldots, p_L)$. The first order conditions for an optimum of the economy $\tilde{\mathcal{E}}$ give the following characterization of a p-optimal allocation for $\mathcal{E}$. It holds that $(\bar{x}^1, \ldots, \bar{x}^N) \in \prod_{i \in \mathcal{N}} X^i$ is a p-optimum if and only if there exist some $q \in \mathbb{R}^L \setminus \{0\}$ and

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$\alpha \in \mathbb{R}^N_+$ such that

$$(\partial_{x^i_l} u^i(\bar{x}^i) - p_l \partial_{x^i_0} u^i(\bar{x}^i))_{l=1,\ldots,L} = \alpha^i q.$$

As we have seen before, at any Drèze equilibrium allocation $\bar{x}$:

$$\partial_{x^i_l} u^i(\bar{x}^i) - p_l \partial_{x^i_0} u^i(\bar{x}^i) = -\underline{\mu}^i_l + \bar{\mu}^i_l,$$

for some non-negative real $\underline{\mu}^i_l, \bar{\mu}^i_l$ such that $\underline{\mu}^i_l \bar{\mu}^i_l = 0$. Let $\mathcal{N}^C \subseteq \mathcal{N}$ be the set of all constrained households, given a Drèze equilibrium $(\bar{x}^1, \ldots, \bar{x}^N)$. Then $\alpha^i = 0$ implies that household i consumes its most preferred element of the set $\{x^i \in X^i | px^i = pw^i\}$. For each $i \in \mathcal{N}^C$, $\alpha^i \neq 0$. Therefore, if a Drèze equilibrium is p-optimal, then

$$q_l = 0 \Rightarrow \underline{\mu}^i_l = \bar{\mu}^i_l = 0, \quad \forall i \in \mathcal{N}^C,$$

$$q_l > 0 \Rightarrow \bar{\mu}^i_l > 0, \quad \forall i \in \mathcal{N}^C,$$

$$q_l < 0 \Rightarrow \underline{\mu}^i_l > 0, \quad \forall i \in \mathcal{N}^C.$$

The vector q is also called a vector of coupons prices in the literature, see Drèze and Müller (1980). Note the one to one correspondence between the side of rationing and the sign of a component of the coupons price vector for p-optimal Drèze equilibria.

The above allows us to formulate a necessary condition for a Drèze equilibrium to be p-optimal. We call this condition the separating property.

Proposition 6.1. *If a Drèze equilibrium at prices $p \in P$ of an economy $\mathcal{E}$ is p-optimal, then every constrained household is constrained in every constrained market.*

It is common in the literature to assume that the households' effective demands are observable, i.e. the household's demand for a commodity when taking into account the rationing schemes on all markets, except on the market of that commodity. A household is constrained in a market if and only if its effective demand is different from its demand. This makes the separating property quite powerful since it is stated in observable data only. Whenever there are two households that face constraints, but in different markets, constrained suboptimality is the case. The separating property is a very strong requirement, so very stringent conditions are needed to achieve constrained optimality.

The separating property is trivially satisfied if $L = 1$, or if there is only one constrained household. The first case has been analyzed in Section 5, where it has been concluded that constrained optimality results in the case with two commodities. In that case the separating property is not only a necessary condition for constrained optimality, but also a sufficient condition.

We already argued that the separating condition is strong, and if satisfied, only a necessary condition. The next result gives conditions for the separating condition to be sufficient for p-optimality.

Theorem 6.2. *Any Drèze equilibrium at prices p of an economy $\mathcal{E}$ with the number of constrained households or the number of constrained markets less than or equal to one, is p-optimal.*

Proof. In the case where the number of constrained households or the number of constrained markets equals zero, the Drèze equilibrium corresponds to a competitive equilibrium allocation, so optimality and therefore p-optimality follows.

Suppose the number of constrained households equals one. Then define $\alpha^i = 1$ and $q_l = -\underline{\mu}^i_l + \bar{\mu}^i_l$, $l = 1, \ldots, L$, with $\underline{\mu}^i_l, \bar{\mu}^i_l$, the Lagrange multipliers corresponding to the rationing constraints of the constrained household i. It follows that the condition for p-optimality is satisfied.

Suppose the number of constrained markets equals one, say market l is constrained. Then define $\alpha^i = -\underline{\mu}^i_l + \bar{\mu}^i_l$, $i \in \mathcal{N}$, and define q to be the l th unit vector in $\mathbb{R}^L$. Again, it follows that the condition for p-optimality is satisfied. □

Our final claim is that under weak conditions, the separating property is typically not a sufficient condition for constrained optimality. More precisely, Drèze equilibria for which the set of constrained households $\mathcal{N}^C$ and the set of constrained markets $\mathcal{L}^C$ consist of more than one element each, are generically not p-optimal.

Theorem 6.3. *Fix any price system $p \in P$ and utility functions $u^1, \ldots, u^N$ satisfying Assumption A2. There is an open set of full Lebesgue measure of initial endowments in $\mathbb{R}^{N(L+1)}_{++}$ such that every Drèze equilibrium at prices p of the economy $\mathcal{E}$ with the number of constrained households and the number of constrained markets greater than or equal to two, is not p-optimal.*

Proof. It is helpful to introduce a vector r that describes the state of the markets. The vector r is an element of

$$R = \{r \in \mathbb{R}^L | r_l = -1, 0, \text{or} 1\},$$

where $r_l = -1$ if there is supply rationing in market l, $r_l = 0$ if there is no rationing in market l, and $r_l = 1$ if there is demand rationing in market l. We also introduce a vector s that describes whether a household is rationed or not. The vector s is an

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element of

$$S = \{s \in \mathbb{R}^N | s^i = 0 \text{ or } 1\},$$

where $s^i = 1$ if and only if i belongs to the set of constrained households $\mathcal{N}^C$.

Denote by $c_l^i(x^i)$ the derivative with respect to x_l^i of the transformed utility function $\tilde{u}^i$ we used before:

$$c_l^i(x^i) = \partial_{x_l^i} u^i(x^i) - p_l \partial_{x_0^i} u^i(x^i).$$

We know from Lemma 3.2 that for any Drèze equilibrium $(\bar{x}^1, \ldots, \bar{x}^N) \in \prod_{i \in \mathcal{N}} X^i$:

$$c_l^i(\bar{x}^i) = -\underline{\mu}_l^i + \bar{\mu}_l^i,$$

for some non-negative $\underline{\mu}_l^i, \bar{\mu}_l^i$ — the Lagrange multipliers corresponding to the rationing constraints. If a Drèze equilibrium $\bar{x}$ is p-optimal, then for any $l, l' \in \mathcal{L}, i, i' \in \mathcal{N}$:

$$\begin{vmatrix} c_l^i(\bar{x}^i) & c_l^{i'}(\bar{x}^{i'}) \\ c_{l'}^i(\bar{x}^i) & c_{l'}^{i'}(\bar{x}^{i'}) \end{vmatrix} = \begin{vmatrix} -\underline{\mu}_l^i + \bar{\mu}_l^i & -\underline{\mu}_l^{i'} + \bar{\mu}_l^{i'} \\ -\underline{\mu}_{l'}^i + \bar{\mu}_{l'}^i & -\underline{\mu}_{l'}^{i'} + \bar{\mu}_{l'}^{i'} \end{vmatrix} = \begin{vmatrix} \alpha^i q_l & \alpha^{i'} q_l \\ \alpha^i q_{l'} & \alpha^{i'} q_{l'} \end{vmatrix} = 0. \quad (1)$$

For given $r \in R$ and $s \in S$, consider the sets:

$$M_{rs} = \{(\mu^i)_{i \in \mathcal{N}^C} \in \mathbb{R}^{|\mathcal{N}^C||\mathcal{L}^C|} | \quad \mu_l^i < 0 \text{ if } r_l = -1, \quad \mu_l^i > 0 \text{ if } r_l = 1\},$$
$$Z_r = \{z \in \mathbb{R}^{|\mathcal{L}^C|} | \quad z_l < \varepsilon \text{ if } r_l = -1, \quad z_l > -\varepsilon \text{ if } r_l = 1\},$$

where ε is some given positive number.

By the Kuhn-Tucker theorem, if $(x^1, \ldots, x^N)$ is a Drèze equilibrium that satisfies the separating property, then there exists $r \in R$, $s \in S$, $\lambda \in \mathbb{R}_{++}^N$, $\mu \in M_{rs}$, and $z \in Z_r$ such that[2]

$$px^i - pw^i = 0, \quad i \in \mathcal{N}, \quad (2)$$

$$\partial_{x_0^i} u^i(x^i) - \lambda^i = 0, \quad i \in \mathcal{N}, \quad (3)$$

$$\partial_{x_l^i} u^i(x^i) - \lambda^i p_l - \mu_l^i = 0, \quad i \in \mathcal{N}, \quad l \in \mathcal{L} \setminus \{0\}, \quad (4)$$

$$\sum_{i \in \mathcal{N}} (x_l^i - w_l^i) = 0, \quad l \in \mathcal{L} \setminus \{0\}, \quad (5)$$

$$x_l^i - w_l^i - z_l = 0, \quad i \in \mathcal{N}^C, \quad l \in \mathcal{L}^C. \quad (6)$$

The number of unknowns in this system equals $N(L+2) + |\mathcal{N}^C||\mathcal{L}^C| + |\mathcal{L}^C|$, which is less than the number of equations $N(L+2) + |\mathcal{N}^C||\mathcal{L}^C| + L$, or is equal to it if $|\mathcal{L}^C| = L$. Since there is a finite number of constrained markets and households, there is a finite number of such systems. Since a finite intersection of open sets of full Lebesgue measure is open and of full Lebesgue measure, it is enough to restrict attention to an arbitrary fixed r and s.

Suppose that $(x^1, \ldots, x^N)$ is a p-optimal Drèze equilibrium. Without loss of generality, $\{1, 2\} \subseteq \mathcal{N}^C$ and $\{1, 2\} \subseteq \mathcal{L}^C$. By the separating property and the equation in determinants derived before:

$$\mu_1^1 \mu_2^2 - \mu_2^1 \mu_1^2 = 0. \quad (7)$$

Thus, p-optimal Drèze equilibria satisfy a system of n equations, where

$$n = (N+1)(L+2) + |\mathcal{N}^C||\mathcal{L}^C| - 1,$$

which is at least one more than there are unknowns.

Let $\psi(w, x, z, \lambda, \mu)$ be the function defined as the left-hand side of the Eqs. (2)–(7). It is defined on $\prod_{i \in \mathcal{N}} X^i \times \prod_{i \in \mathcal{N}} X^i \times Z_r \times \mathbb{R}_{++}^N \times M_{rs}$ and takes its values in $\mathbb{R}^{(N+1)(L+2)+|\mathcal{N}^C||\mathcal{L}^C|-1}$. The key element of the proof is the fact that ψ is transversal to the origin, i.e. whenever $\psi(w, x, z, \lambda, \mu) = 0$, its Jacobian is of full rank.

Suppose that there exists $y \in \mathbb{R}^n$ such that

$$y^\top \partial \psi(\cdot) = 0.$$

[2] Notice that a Drèze equilibrium always leads to a solution to the system of equations. The other way around is not necessarily true, since non-binding inequality constraints have been omitted, and the definition of Z_r implies that a limited amount (ε) of forced trading is not excluded in a solution to the system of equations.

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We write $y = (y_1, \ldots, y_6)$, where $y_1 \in \mathbb{R}^N$, $y_2 \in \mathbb{R}^N$, $y_3 \in \mathbb{R}^{NL}$, $y_4 \in \mathbb{R}^L$, $y_5 \in \mathbb{R}^{|\mathcal{N}^C||\mathcal{L}^C|}$, and $y_6 \in \mathbb{R}$. Then, in particular,

$$y^\top \partial_{w_0^i} \psi(\cdot) = -p_0 y_{1,i} = 0,$$

so $y_1 = 0$. If $l \in \mathcal{L} \setminus \mathcal{L}^C$, then taking into account the previous expression one gets:

$$y^\top \partial_{w_l^1} \psi(\cdot) = -y_{4,l} = 0.$$

For $l \in \mathcal{L}^C$ we have

$$\begin{array}{llll} y^\top \partial_{w_l^i} \psi(\cdot) & = -y_{4,l} - y_{5,i,l} & = 0, & \text{for } i \in \mathcal{N}^C \\ y^\top \partial_{w_l^i} \psi(\cdot) & = -y_{4,l} & = 0, & \text{for } i \in \mathcal{N} \setminus \mathcal{N}^C, \end{array}$$

and

$$y^\top \partial_{z_l} \psi(\cdot) = -\sum_{i \in \mathcal{N}^C} y_{5,i,l} = 0.$$

Thus, $\sum_{i \in \mathcal{N}^C} y_{5,i,l} = |\mathcal{N}^C| y_{4,l}$, so $y_{4,l} = 0$, $l \in \mathcal{L}^C$. Therefore, $y_{5,i,l} = 0$, $i \in \mathcal{N}^C$, $l \in \mathcal{N}^C$. Moreover,

$$y^\top \partial_{\mu_1^1} \psi(\cdot) = y_6 \mu_2^2 = 0,$$

so $y_6 = 0$. To complete the proof of regularity it is sufficient to show that the matrix:

$$(\partial^2 u^i(\bar{x}^i), -p^\top)$$

has a full row rank for every $i \in \mathcal{N}$. This follows from the differentiable strict quasi-concavity of the utility function, Proposition 2.6.4 of Mas-Colell (1985), and the possibility to cover a consumption set by a countable number of compacts.

By the Transversality Theorem, $\partial_{(x,z,\lambda,\mu)} \psi^w(x, z, \lambda, \mu)$ has full rank for almost all $w \in \mathbb{R}_{++}^{N(L+1)}$ if $\psi^w(x, z, \lambda, \mu) = 0$, where $\psi^w(x, z, \lambda, \mu) = \psi(w, x, z, \lambda, \mu)$. Therefore, generically in w, the inverse image of $\{0\}$ has the same co-dimension as zero, which implies that $\psi^w(x, z, \lambda, \mu) = 0$ has no solution. We conclude that for a set of endowments of full Lebesgue measure, any Drèze equilibrium is constrained suboptimal.

Denote by S the set $\prod_{i \in \mathcal{N}} X^i \times Z_r \times \mathbb{R}_{++}^N \times M_{rs}$. To show that we can choose the set of initial endowments of full Lebesgue measure for which Drèze equilibria are constrained suboptimal to be open, consider the set Σ of all $(w, x, z, \lambda, \mu) \in \mathbb{R}_{++}^{N(L+1)} \times S$ such that $\psi(w, x, z, \lambda, \mu) = 0$. This set is closed by continuity of ψ. Moreover, it is not difficult to see that the set $\{(w, x, z, \lambda, \mu) \in \Sigma \mid w \in K\}$ is compact for any compact subset K of $\mathbb{R}_{++}^{N(L+1)}$. The latter means that the natural projection function $\pi : \Sigma \to \mathbb{R}_{++}^{N(L+1)}$ that maps (w, x, z, λ, μ) to w is proper. Therefore, the set of all w for which the conclusion of the theorem does not hold, is closed as the image of a closed set by a proper function. Its complement is open and, as has been shown before, contains a set of full Lebesgue measure. □

The condition that the number of constrained markets is greater than or equal to two may be omitted from the statement of Theorem 6.3, since it is a generic property when $L > 1$. The proof of this fact goes along the same lines as the proof of the theorem above. Thus, we have the following corollary.

Corollary 6.4. *Fix any price system $p \in P$ and utility functions $u^1, \ldots, u^N$ satisfying Assumption A2. There is an open set of full Lebesgue measure of initial endowments in $\mathbb{R}_{++}^{N(L+1)}$ such that every Drèze equilibrium at prices p of the economy $\mathcal{E}$ with the number of constrained households greater than or equal to two, is not p-optimal.*

It is not possible to claim that the number of constrained households is generically greater than one. We show that for any tuple of utility functions $u^1, \ldots, u^N$ satisfying Assumption A2, there is an open set of initial endowments $\Omega \subset \mathbb{R}_{++}^{N(L+1)}$ such that for every $w \in \Omega$ there is a Drèze equilibrium with only one constrained household. The example below is constructed in such a way that the constrained household is constrained in its supply in all markets, whereas all other households have small net demands for all non-numeraire commodities.

Consider any tuple of utility functions $u^1, \ldots, u^N$ satisfying Assumption A2 and fix an arbitrary price system $p \in P$. Let $\bar{x}^i$ be the notional demand of household i at prices p when it has initial endowments e, where e is the vector of all ones in $\mathbb{R}^{L+1}$. Pick initial endowments for household 1 and a rationing scheme $(\underline{z}, \bar{z})$ such that household 1, at prices p and rationing scheme $(\underline{z}, \bar{z})$, is constrained in its supply in each market for non-numeraire commodities and $-\underline{z}$ is smaller than $\bar{x}^i$ for $i = 2, \ldots, N$. To achieve this, one may take $w^1 \in \{w \in X^1 \,|\, pw = pe\}$ such that $w_{-0}^1 \gg \bar{x}_{-0}^1$, so the notional demand of household 1 at prices p equals $\bar{x}^1$, and household 1 prefers to supply all non-numeraire commodities. Take $\underline{z}_1^1 = \tilde{x}_1^1 - w_1^1 + \varepsilon^1$, with ε^1 a small positive number. By continuity, the demand $\tilde{x}^1$ of household 1 when taking $\underline{z}_1$ into account is close to $\bar{x}^1$, in particular all non-numeraire commodities are still supplied by household 1, and rationing in the market for commodity 1 is binding. Take $\underline{z}_2 = \tilde{x}_2^1 - w_2^1 + \varepsilon^2$, with ε^2 a positive number that is small enough for rationing in the market for commodity 1 to remain binding. This construction is repeated until the rationing scheme $\underline{z}$ is obtained that induces rationing on the supply of household 1 in the markets for all non-numeraire commodities. By taking w^1 sufficiently close to $\bar{x}^1$, the requirement that

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$-\underline{z}$ be smaller than $\bar{x}^i$, $i = 2, \ldots, N$, can be fulfilled as well. The rationing scheme $\bar{z}$ is chosen as never to affect the choice of any household.

For $i = 2, \ldots, N$, initial endowments are taken such that household i is on the short side of each market,

$$w^i = \bar{x}^i - \begin{pmatrix} \sum_{l=1}^{L} p_l \underline{z}_l/(N-1) \\ -\underline{z}_1/(N-1) \\ \vdots \\ -\underline{z}_L/(N-1) \end{pmatrix}.$$

Since all households, but household 1, get their most preferred commodity bundle at prices p, it follows that $(d^1(\underline{z}, \bar{z}, p), \bar{x}^2, \ldots, \bar{x}^N)$ is a p-optimal Drèze equilibrium. If we slightly perturb the initial endowments, total net demand of households excluding household 1 changes slightly, and a Drèze equilibrium is obtained by rationing the supply of household 1 by this amount. All other households remain unconstrained. The property of p-optimality is kept. It follows that there is an open set of initial endowments with p-optimal Drèze equilibria.

A natural explanation for the existence of price regulations or price rigidities is that they are an instrument that leads to Pareto improvements in a world with imperfections (see Polemarchakis, 1979; Drèze and Gollier, 1993; Drèze, 2001; Herings and Polemarchakis, 2005). This may seem at odds with the statement of Theorem 6.3, where it is shown that price rigidities do not even lead to constrained optimal allocations. In a world with imperfections, however, competitive equilibrium allocations fail to be constrained optimal too, and creates scope for Pareto improvements by means of price regulation. One should expect Theorem 6.3 to remain valid when imperfections are added to the model. This is hardly surprising, as it is more difficult to show constrained suboptimality in a world without imperfections. As a consequence, whenever in a world with imperfections, a competitive allocation is Pareto improved by a Drèze equilibrium allocation, the latter is subject to further Pareto improvements. Below we discuss how such improvements can be implemented.

To what extend does a social planner have to intervene in the economy in order to improve upon a constrained equilibrium? Suppose that $\bar{x}$ is a Drèze equilibrium. One way for a planner to change the situation would be to endow each household with a strictly positive amount of coupons and allow them to trade under price system p and any possible coupon prices they choose. By virtue of the results of Drèze and Müller (1980), the resulting allocation will be a p-optimal coupons equilibrium Pareto superior to $\bar{x}$.

As a careful analysis of the proof of Theorem 6.3 suggests, a planner can also improve the situation by imposing a trade that involves only two constraint agents and two constrained markets of non-numeraire goods. Namely, it was shown that condition (1) is generically violated for any pair of constrained households and any pair of constrained markets. Therefore for any $i_1, i_2 \in \mathcal{N}$, $l_1, l_2 \in \mathcal{L} \setminus \{0\}$, a system of inequalities:

$$\begin{pmatrix} c^{i_1}_{l_1}(\bar{x}^{i_1}) & c^{i_1}_{l_2}(\bar{x}^{i_1}) \\ -c^{i_2}_{l_1}(\bar{x}^{i_2}) & -c^{i_2}_{l_2}(\bar{x}^{i_2}) \end{pmatrix} \begin{pmatrix} z_{l_1} \\ z_{l_2} \end{pmatrix} \gg 0$$

generically has a solution whenever households i_1 and i_2 are constrained in markets l_1 and l_2. It is clear that (z_{l_1}, z_{l_2}) can be chosen small enough to guarantee that a consumption bundle defined by

$$y^{i_1}_l = \begin{cases} \bar{x}^{i_1}_0 - p_{l_1} z_{l_1} - p_{l_2} z_{l_2}, \\ \bar{x}^{i_1}_{l_1} + z_{l_1}, \\ \bar{x}^{i_1}_{l_2} + z_{l_2}, \\ \bar{x}^{i_1}_l, \quad l \notin \{0, l_1, l_2\}, \end{cases}$$

$$y^{i_2}_l = \begin{cases} \bar{x}^{i_2}_0 + p_{l_1} z_{l_1} + p_{l_2} z_{l_2}, \\ \bar{x}^{i_2}_{l_1} - z_{l_1}, \\ \bar{x}^{i_2}_{l_2} - z_{l_2}, \\ \bar{x}^{i_2}_l, \quad l \notin \{0, l_1, l_2\}, \end{cases}$$

and $y^i_l = \bar{x}^i_l$ for $i \notin \{i_1, i_2\}$, is feasible and Pareto superior to $\bar{x}$. Thus we have the following proposition.

Proposition 6.5. *Fix any price system $p \in P$ and utility functions $u^1, \ldots, u^N$ satisfying Assumption A2. There is an open set of full Lebesgue measure of initial endowments in $\mathbb{R}^{N(L+1)}_{++}$ such that for every Drèze equilibrium $\bar{x}$ with at least two constrained households there exists a feasible allocation y such that, for all i, $py^i = p\bar{x}^i$, $|\{(i, l) \in \mathcal{N} \times \mathcal{L} | y^i_l \neq \bar{x}^i_l\}| \leq 6$, and y Pareto dominates $\bar{x}$.*

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7. Flexible prices

So far the analysis has been carried our for the case where the prices of all commodities are fixed. What we have in mind are prices which are temporarily fixed and against which trade takes place. The requirement that these trades are compatible with a Drèze equilibrium makes sure that resulting allocations are consistent.

In a more general case, price rigidities only exist on some markets, whereas other markets are cleared in the usual way by price adjustments, without reliance on rationing. We follow the set-up in Citanna et al. (2001), and assume there is a non-empty set of commodities $\mathcal{C} \subset \mathcal{L} \setminus \{0\}$ with cardinality C for which prices are fixed and rationing may occur, whereas the markets of commodities in $\mathcal{L} \setminus \mathcal{C}$ are cleared without rationing. Commodity 0 continues to be the numeraire commodity with price equal to 1. Fixed prices belong to the set $P^{\mathcal{C}} = \mathbb{R}^{C}_{++}$. When a set like $\mathcal{C}$ or $\mathcal{L}$ appears as a superscript of a vector, only the corresponding components of the vector are included. We now consider an exchange economy $\mathcal{E}^* =< \mathcal{N}, \mathcal{L}, \mathcal{C}, \{X^i, u^i, w^i\}_{i\in\mathcal{N}} >$.

Following Drèze (1975), we introduce a Drèze equilibrium of the economy $\mathcal{E}^*$ at prices $p^{\mathcal{C}} \in P^{\mathcal{C}}$.

Definition 7.1. A Drèze equilibrium at prices $p^{\mathcal{C}} \in P^{\mathcal{C}}$ of an economy $\mathcal{E}^*$ is an allocation $(\bar{x}^1, \ldots, \bar{x}^N) \in \prod_{i\in\mathcal{N}} X^i$ such that there exists $(\underline{z}, \bar{z}) \in \mathbb{R}^{C}_{-} \times \mathbb{R}^{C}_{+}$ and a price system $\bar{p} \in P$ satisfying the following conditions:

(i) for all $i \in \mathcal{N}$, $\bar{x}^i$ maximizes u^i on $B^i(\underline{z}, \bar{z}, \bar{p})$;
(ii) $\sum_{i\in\mathcal{N}} \bar{x}^i = \sum_{i\in\mathcal{N}} w^i$;
(iii) for every $l \in \mathcal{C}$,

$$\bar{x}^{i'}_l - w^{i'}_l = \bar{z}_l \text{ for some } i' \in \mathcal{N} \text{ implies } \bar{x}^i_l - w^i_l > \underline{z}_l \text{ for all } i \in \mathcal{N},$$
$$\bar{x}^{i'}_l - w^{i'}_l = \underline{z}_l \text{ for some } i' \in \mathcal{N} \text{ implies } \bar{x}^i_l - w^i_l < \bar{z}_l \text{ for all } i \in \mathcal{N}.$$

(iv) $\bar{p}^{\mathcal{C}} = p^{\mathcal{C}}$.

Compared to Definition 2.1, prices of commodities outside $\mathcal{C}$ are fully flexible. As a natural complement of price flexibility, there is no rationing on the markets of these commodities.

Many of the results derived in the previous section remain useful. The reason is that a Drèze equilibrium $(\bar{x}^1, \ldots, \bar{x}^N)$ at prices $p^{\mathcal{C}} \in P^{\mathcal{C}}$ of an economy $\mathcal{E}^*$ for a price system $\bar{p}$ and a rationing scheme $(\underline{z}^{\mathcal{C}}, \bar{z}^{\mathcal{C}})$ according to Definition 7.1, induces a Drèze equilibrium $(\bar{x}^1, \ldots, \bar{x}^N)$ at prices $\bar{p} \in P$ of an economy $\mathcal{E}$ for a rationing scheme $(\underline{z}, \bar{z})$ according to Definition 2.1. Here, the components of $\underline{z}, \bar{z}$ outside $\mathcal{C}$ are taken large enough in absolute value as not to affect the decisions of any of the households.

We continue by proving generic suboptimality of equilibrium in this extended setting. The main difference with Section 3 is that a perturbation in the endowments will now affect the prices of commodities outside $\mathcal{C}$.

Theorem 7.2. *Fix any price system $\bar{p}^{\mathcal{C}} \in P^{\mathcal{C}}$ and utility functions $u^1, \ldots, u^N$ satisfying Assumption A2. Then there is an open set of full Lebesgue measure of initial endowments in $\mathbb{R}^{N(L+1)}_{++}$ such that every Drèze equilibrium at prices $\bar{p}^{\mathcal{C}}$ of the economy $\mathcal{E}$ is suboptimal.*

Proof. Since Drèze equilibria at prices $\bar{p}^{\mathcal{C}}$ are Drèze equilibria at prices $\bar{p} \in P$ for a suitably chosen $\bar{p}$, Proposition 3.3 yields that an optimal Drèze equilibrium at prices $\bar{p}^{\mathcal{C}}$ corresponds to a competitive equilibrium allocation.

It follows from the results in Laroque (1978) that for an open set of full Lebesgue measure of initial endowments, for every competitive equilibrium allocation x^*, $x^{*i}_l - w^i_l \neq 0$, for every household i and every commodity l. Therefore, using Lemma 3.2, generically in initial endowments, for $l \in \mathcal{C}$, cases where all $\underline{\mu}^i_l > 0$ and all $\bar{\mu}^i_l > 0$ are excluded. Generically in endowments, an optimal Drèze equilibrium consists of competitive equilibrium prices and a competitive equilibrium allocation. To complete the proof we need to show that for generic w a price system $\bar{p}$ supporting a Drèze equilibrium is not competitive. Let $z(p, w)$ denote aggregate excess demand at prices p and endowments $w = (w^1, \ldots, w^N)$. Let $F_l(p, w)$ be equal to $z_l(p, w)$ for $l = 1, \ldots, L$, and define $F_{L+1}(p, w) = p_l - \bar{p}_l$ for an arbitrarily chosen l in $\mathcal{C}$. The remainder of the proof is identical to the one of Theorem 3.4. □

Theorem 7.2 provides a rigorous statement of the Folk Theorem on the generic suboptimality of equilibria when prices of some commodities are non-competitive.

The next step involves the analysis of constrained optimality. In the spirit of Definition 4.2, we define constrained optimality of a Drèze equilibrium as follows.

Definition 7.3. A Drèze equilibrium $(x^1, \ldots, x^N) \in \prod_{i\in\mathcal{N}} X^i$ at prices $p^{\mathcal{C}}$ of an economy $\mathcal{E}^*$ for a rationing scheme $(\underline{z}, \bar{z})$ and prices $\bar{p}$ is constrained optimal if there is no allocation $(y^1, \ldots, y^N) \in \prod_{i\in\mathcal{N}} X^i$ such that

(i) $\forall i \in \mathcal{N}$ $\bar{p}y^i = \bar{p}x^i$,

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(ii) $\sum_{i\in\mathcal{N}} y^i = \sum_{i\in\mathcal{N}} w^i$,
(iii) $\forall i \in \mathcal{N} u^i(y^i) \geq u^i(x^i)$ with strict inequality for at least one $i' \in \mathcal{N}$.

In Sections 5 and 6 we distinguished between the cases $L = 1$ and $L \geq 2$. Now, we have to treat the cases $C = 1$ and $C \geq 2$ separately. Notice that the only restriction imposed upon L by $C = 1$ is that L is greater than or equal to one. The following result is therefore a substantial generalization of Proposition 5.1.

Theorem 7.4. *If $C = 1$, then a Drèze equilibrium $(\bar{x}^1, \ldots, \bar{x}^N)$ at prices $p^{\mathcal{C}} \in P^{\mathcal{C}}$ of an economy $\mathcal{E}^*$ is constrained optimal.*

Proof. Let $(\bar{x}^1, \ldots, \bar{x}^N)$ be a Drèze equilibrium for a rationing scheme $(\underline{z}, \bar{z})$ and a price system $\bar{p}$.

If no household is constrained, then $(\bar{x}^1, \ldots, \bar{x}^N)$ is a competitive equilibrium allocation, so both optimal and constrained optimal.

If some household is constrained in its supply, then none of the households is constrained in its demand by virtue of Condition (iii) of Definition 7.1. Suppose $(y^1, \ldots, y^N)$ is an allocation satisfying Conditions (i)–(iii) in Definition 7.3.

Since $C = 1$, a household i that is not constrained in its supply, faces no rationing constraints at all, and chooses a best consumption bundle in its budget set. There is no x^i such that $px^i = p\bar{x}^i$ and $u^i(x^i) > u^i(\bar{x}^i)$ and, using strict convexity of u^i, $u^i(y^i) \geq u^i(\bar{x}^i)$ implies $y^i = \bar{x}^i$.

When household i is constrained in its supply, a similar revealed preference argument can be used to show that $u^i(y^i) \geq u^i(\bar{x}^i)$ implies $y^i_l \leq \bar{x}^i_l$ and $u^i(y^i) > u^i(\bar{x}^i)$ implies $y^i_l < \bar{x}^i_l$, where $l \in \mathcal{C}$. We therefore derive, for $l \in \mathcal{C}$:

$$\sum_{i\in\mathcal{N}} w^i_l = \sum_{i\in\mathcal{N}} y^i_l < \sum_{i\in\mathcal{N}} \bar{x}^i_l = \sum_{i\in\mathcal{N}} w^i_l,$$

a contradiction. Consequently, there is no allocation $(y^1, \ldots, y^N)$ satisfying Conditions (i)–(iii) in Definition 7.3.

The argument for the case where some household is constrained in its demand is analogous. □

Theorem 7.4 replicates the constrained optimality result of Proposition 5.1 for the case where there is only one market with rationing constraints. Contrary to the case $L = 1$, the equilibrium can no longer be guaranteed to be unique. We provided an elementary proof of Theorem 7.4. We would like to remark that it also follows as a corollary to Theorem 6.2.

Though Theorem 7.1 is a considerable extension of Proposition 5.1, and may have applicability in macro-economic contexts, where a natural assumption only admits rationing in the labor market, the case $C = 1$ remains restrictive in a general equilibrium set-up. Even when labor would be the only commodity subject to price rigidities, C would exceed 1 by far in a general equilibrium model, since there are many types of labor, supplied at a variety of dates, and contingent on all kinds of events. The more important case, therefore, remains $C \geq 2$, which is studied next.

Obviously, as it involved no genericity arguments, the separating property of Proposition 6.1 remains valid, and the following result is obtained as a corollary.

Corollary 7.5. *If a Drèze equilibrium at prices $p^{\mathcal{C}} \in P^{\mathcal{C}}$ of an economy $\mathcal{E}^*$ is constrained optimal, then every constrained household is constrained in every constrained market.*

Also in an environment with price flexibility, the separating condition is strong, and would be violated if two households, providing different types of labor services, are both experiencing supply constraints. Moreover, the separating property is only a necessary condition, and not a sufficient one. When in equilibrium there is only one constrained household, the separating property is trivially satisfied, and using similar arguments as in the proof of either Theorem 6.2 or Theorem 7.4, constrained optimality can be shown to hold. For the case of two or more constrained households, we obtain the following result.

Theorem 7.6. *Fix any price system $p^{\mathcal{C}} \in P^{\mathcal{C}}$ and utility functions $u^1, \ldots, u^N$ satisfying Assumption A2. There is an open set of full Lebesgue measure of initial endowments in $\mathbb{R}^{N(L+1)}_{++}$ such that every Drèze equilibrium at prices $p^{\mathcal{C}}$ of the economy $\mathcal{E}$ with the number of constrained households greater than or equal to two, is not constrained optimal.*

Proof. A standard genericity argument can be used to show that when $C \geq 2$, there are typically C constrained markets. We can now proceed along the lines of the proof of Theorem 6.3, where we impose the vector r to be an element of

$$R = \{r \in \mathbb{R}^C | r_l = -1 \text{ or } 1\},$$

s of

$$S = \{s \in \mathbb{R}^N | s^i = 0 \text{ or } 1\}.$$

and replace $\mathcal{L}^C$ by $\mathcal{C}$. □

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8. Conclusions

Notwithstanding the strong conviction of most economists that trade at non-competitive prices has detrimental welfare consequences, it is not based on foundations derived with equal rigor as the first and second welfare theorems. This paper provides these foundations. We show that the Folk Theorem holds that equilibria are typically suboptimal when trade at non-competitive prices occurs.

The more appropriate question to answer is whether equilibria are perhaps not even constrained optimal. In this paper we formalized the notion of constrained optimality as optimality among allocations where budget constraints of all households at the non-competitive prices are met. A necessary condition for constrained optimality to prevail is the separability condition that all constrained households be constrained in all constrained markets. This condition is of interest in itself as it is formulated in terms that are observable. If the number of markets subject to price rigidities is less than or equal to one, then this necessary condition is always satisfied. We show that in this case it is also a sufficient condition, so constrained optimality holds. Also the extremely restrictive case where there is at most one constrained household, leads to constrained optimality of equilibrium. For the most important and realistic scenario with two or more markets subject to price rigidities, and two or more constrained households, equilibria are typically constrained suboptimal.

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