M.A. Kahn and N.C. Yannelis (eds.), *Equilibrium Theory in Infinite Dimensional Spaces,* Springer-Verlag, Berlin, *etc.,* 1991. Pp. x+435.

The book *Equilibrium Theory in Infinite Dimensional Spaces* consists of nineteen until now unpublished contributions which treat rather diverse subjects. The contributions have in common that they are dealing with infinite dimensional spaces, interpreted either as a commodity space or as a strategy space. Papers dealing with infinite dimensional spaces often use very technical mathematics. The papers in this book are no exception to this observation. In the literature dealing with infinite dimensional spaces it is sometimes even difficult to attach an economic interpretation to the models discussed and the results obtained. Nevertheless, research in this area is extremely important for the development of economic theory. If one wants to model things like for example time or location, one has to use infinite dimensional spaces. Other examples where infinite dimensional spaces are needed are cases where the quality of a commodity is selected out of a continuum of possibilities, or where there is an infinite number of possible states of the world. There are also cases where models with an infinite number of agents are needed. The most well-known example is that in a world with a finite number of commodities, presence of an infinite number of agents is the only way to justify price taking behaviour. This gives some motivation why economists not interested in mathematics should read this book too, and get acquainted with its most important results.

The first part of the book, called 'Mathematical Background,' consists of two papers of N.C. Yannelis. In a finite economy price-taking behaviour of agents is not sensible. Therefore economies with atomless measure spaces of agents are considered. In such an economy the total demand is obtained by integrating a set valued function. If one wants to allow for an infinite dimensional commodity space, one has to integrate a Banach-valued set function. The properties of such an integral are the subject of the paper 'Integration of Banach-Valued Correspondences.' The second paper 'Set-Valued Functions of Two Variables in Economic Theory' gives a survey of properties of set-valued functions of two variables like the existence of continuous selections, measurable selections, Carathéodory-type selections, the existence of random fixed points and the existence of random maximal elements.

The second part of the book, called 'Equilibria, Core, and Pareto Optimality,' consists of three papers. In his paper 'A Theorem on the Existence of Competitive Equilibria in a Market with a Finite Number of Agents and Whose Commodity Space is L_∞,' T.F. Bewley presents an equilibrium existence proof where the dimension of the sets used in the proof does not depend on the dimension of the commodity space, but on the number of consumers. This type of proof might be extended in order to get a very general theorem on the existence of competitive equilibria for economies with an infinite dimensional commodity space. The second paper, 'The Core of an Economy Without Ordered Preferences' of N.C. Yannelis, gives conditions for the existence of core allocations and conditions for the existence of individual rational Pareto optimal allocations for exchange economies with a finite number of consumers and an infinite dimensional commodity space. Preferences need not be transitive, complete, monotone or convex. In an appendix an elementary proof of the Knaster-Kuratowski-Mazurkiewicz-Fan theorem is given. The third paper, 'Fundamental Theorems of Welfare Economics in Infinite Dimensional Commodity Spaces' of R.A. Becker,

is a survey of the existence of price supports for Pareto optimal allocations in economies with a finite number of consumers and producers and an infinite dimensional commodity space. Problems arise if the positive cone of the commodity space has an empty interior. To compensate for the lack of interior points, additional assumptions with respect to the preferences have to be made. If these assumptions are not satisfied, approximate price supports have to be considered.

The third part of the book, called 'Core Equivalence,' consists of three papers. In the paper 'A Limit Theorem on the Core of an Economy with a Continuum of Commodities' J.J. Gabszewicz extends the Edgeworth-Scarf-Debreu theorem and allows for the case with an infinite dimensional commodity space. The paper 'An Equivalence Theorem for the Core of an Economy with Commodity Space $L_\infty - \tau(L_\infty, L_1)'$ of J.-F. Mertens deals with the case with a continuum of commodities and in addition an atomless measure space of agents. The paper 'The Principle of Equivalence' of H.H.C. Cheng also considers the equivalence of the core of a pure exchange economy and the set of Walrasian equilibrium allocations. Again there is an infinite number of commodities and an atomless measure space of agents. However, instead of describing the economy by individual consumption constraints, preferences and endowments, a more general coalitional representation is used.

The fourth part of the book is called 'The Existence of an Equilibrium in Economies with a Continuum of Agents.' It contains three papers. In the paper 'A Very Weak Theorem on the Existence of Equilibria in Atomless Economies with Infinitely Many Commodities' of T.F. Bewley an equilibrium existence proof is given. Since the combination of an infinite number of agents and an infinite number of commodities leads to several mathematical difficulties, the assumptions made are rather restrictive. In the paper 'Equilibria in Markets with a Continuum of Agents and Commodities,' M.A. Khan and N.C. Yannelis also give an equilibrium existence proof. The main difference with the paper of T.F. Bewley is that the measure space of agents is not necessarily atomless. The paper 'What is Perfect Competition?' of A. Rustichini and N.C. Yannelis considers a pure exchange economy with a measure space of agents and an infinite dimensional commodity space. A mathematical formulation of the idea of perfect competition, i.e. no individual can influence the price of commodities, is given. If an appropriate assumption with respect to the dimension of the space of agents is made, it is possible to prove the existence of a competitive equilibrium, even without making the convexity assumption on preferences. Sufficient conditions for the existence of a pure strategy Nash equilibrium in games with a continuum of players and an infinite dimensional strategy space are given too.

Part 5 of the book, 'Correlated Equilibria,' consists of three papers. In the paper 'On the Existence of Correlated Equilibria,' N.C. Yannelis and A. Rustichini give sufficient conditions for the existence of correlated equilibria in non-cooperative games with finitely many players and infinitely dimensional strategy sets. Moreover, in case the strategy set is equal to $\mathbb{R}^l$ sufficient conditions are given for the existence of a pure strategy correlated equilibrium. In the paper 'Existence of Correlated Weak Equilibria in Discontinuous Games' of A.S. Nowak sufficient conditions are given for the existence of correlated weak $\varepsilon-$ equilibria for some non-zero-sum discontinuous games with a finite number of players and infinite dimensional strategy spaces. If utility functions are continuous it can be shown that a correlated weak equilibrium exists. The proofs do not use any fixed point argument. In the paper 'Communication Equilibria with Large State Spaces' of K.D. Cotter some

types of correlation equilibria are discussed, differing with respect to the correlation device they use. There is a finite number of players, and players are allowed to have an arbitrary, possibly infinite, set of possible states of information.

Part 6 of the book, 'Games with a Continuum of Players,' contains 3 papers. In the paper 'An Axiomatic Approach to the Efficiency of Non-Cooperative Equilibrium in Economies with a Continuum of Traders' of A. Mas-Colell a non-cooperative game theoretic analysis of general equilibrium models with a continuum of agents and a finite number of commodities is given. In the paper an axiomatization of an abstract trading game is given, which yields the conclusion that Cournot-Nash equilibria of the trading game are Walrasian equilibria of the economy. Moreover, sufficient conditions for the existence of a Cournot-Nash equilibrium distribution of a game are given. In the paper 'On Symmetric Cournot-Nash Equilibrium Distributions in a Finite-Action, Atomless Game' of M.A. Kahn and Y.N. Sun it is shown that for every Cournot-Nash equilibrium of a game there exists an in some sense equivalent symmetric Cournot-Nash equilibrium. In the paper 'Equilibria in Random and Bayesian Games with a Continuum of Players,' E.J. Balder and N.C. Yannelis show the existence of random Nash equilibria for games with a continuum of players and an infinite dimensional strategy space. Preferences of players depend on the random state of nature and are not necessarily transitive or complete. Also the existence of a symmetric Bayesian equilibrium is shown.

Part 7 of the book, 'Sequential Equilibria,' consists of 2 papers. In the paper 'Recursive Utility under Uncertainty' of S.H. Chew and L.G. Epstein an axiomatization of a broad class of utility functions, called recursive utility functions is given. They consider an infinite time horizon and allow for uncertainty. Their axiomatization allows for some degree of separation between intertemporal substitutability and risk aversion. The paper 'Consistency and Continuity of Choice in a Sequence of Spot and Futures Markets' of D.C. Nachman and R.P. Kertz deals with an exchange economy where markets open at each of an infinite sequence of dates. If markets are open it is possible to operate on the spot market and on the market of futures for the next period. Sufficient conditions for consistency and continuity of the choices of the agents are given in the absence of an institutional arrangement for handling bankruptcy.

For a fruitful understanding of the book, knowledge of the basic concepts of functional analysis, topology, and measure theory is required. Each paper gives the relevant mathematical references. Considering the complexity of the material presented, most papers are relatively accessible and very clear. Although some type-writing errors are present in the book, I have not been able to find any mathematical error in any of the papers. The book provides an excellent overview of the state of the art of equilibrium theory in infinite dimensional spaces and can especially be recommended to everyone doing research in this area.

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