

Villanacci, A., L. Carosi, P. Benevieri, and A. Battinelli: *Differential Topology and General Equilibrium with Complete and Incomplete Markets*. 473 pp. Kluwer Academic Publishers, Boston, 2002.

The book provides an excellent overview of what modern general equilibrium theory has to offer. By modern I refer to the fact that time and uncertainty are taken into account in a satisfactory way. All agents in the models considered hold intertemporal objectives. At the same time it is realized that only limited commitments can be made regarding economic activity in the future and that all such activity is subject to uncertainty.

Although the notion general equilibrium is too often interpreted in a rather loose way, the authors characterize the general equilibrium approach rather precisely as being based on three principles: methodological individualism, the maximization principle, and the consistency principle.

Methodological individualism means that ultimately every description of a collective entity should depend only on a description of individuals constituting the collective entity. Collective behavior follows as a consequence of behavior of all individuals involved in the entity.

The maximization principle implies that individual behavior should be formulated as the result of meeting objectives with feasible alternatives. Households maximize intertemporal utility functions subject to a sequence of budget constraints, firms typically are assumed to behave in the benefit of shareholders subject to constraints induced by the available technologies, politicians (though not modeled by the authors) choose within a set of feasible policy options the one that maximizes their objective function.

Where most of the economic literature takes methodological individualism and the maximization principle as its foundations, the general equilibrium approach is distinguished by also taking the consistency principle seriously. Individuals' behavior as induced by the maximization principle should be mutually consistent. Supply and demand should be equal on all markets simultaneously, both financial and real markets, as well current markets as

markets opened in the future. Individuals should optimize on the basis of market clearing prices, including expectations of prices that will clear markets opened in the future.

The importance of a thorough understanding of the idealized model of general equilibrium with time and uncertainty that results from the application of these three principles cannot be overestimated. It constitutes the underpinning of what most of the economic literature is about. It does, however, typically lead to too limited attention for a number of important issues. The consistency principles focuses attention on equilibrium states. This ignores the pressing question of whether an economy out of equilibrium reaches any equilibrium state, if so, which equilibrium state is attained, and how exactly the equilibrating process operates.

The maximization principle typically, though not necessarily, implies fully rational decision making, as the costs of decision making are usually not made explicit. It involves individuals with consistent intertemporal preferences that have common expectations on possible future states of nature, future preferences and availability of future technologies. Individuals are then assumed to be able to compute equilibrium prices, and, moreover, are able to coordinate jointly on one particular system of equilibrium prices.

The authors do not contribute to the solution of these important shortcomings of modern general equilibrium theory. It would be unfair to blame them for this, as this simply reflects the state-of-the-art in the majority of general equilibrium research. It does, however, clearly point out that also in general equilibrium theory important conceptual contributions remain to be made.

The authors do provide an extensive account of the most important issues to be found in the recent general equilibrium literature. They emphasize the role of market incompleteness, which implies that at each date-event there are spot markets for commodities and a limited number of financial markets that serve the role of reshuffling income across future date-events. Securities (or assets) traded on financial markets can be nominal, numeraire

or real. Nominal assets have payoffs in units of account. Numeraire assets have payoffs in units of a particular commodity (basket) like for instance gold. Real assets are more general than nominal assets. Real assets specify arbitrary state-contingent deliveries of commodity baskets. The distinction between different types of securities is crucial, and has major consequences for fundamental issues like existence and efficiency of equilibria.

The authors take the differentiable approach, and as a consequence, make full use of the apparatus of differential topology. They should be congratulated for their effort to present this apparatus in Part I of their book in a way that is as simple and transparent as possible, but without making compromises with regard to the required level of generality. I am not aware of any other text in economics that gives a comparable treatment.

The book is required reading for anyone that would like to go beyond the traditional complete markets general equilibrium model, and that wants to have a deeper understanding of the role played by financial markets. Any book of this kind is necessarily demanding on the reader. Still, this book is accessible to anyone with a solid background in calculus. Although a number of important issues related to the modeling of out-of-equilibrium behavior and of bounded rationality are not covered by this book, it offers a complete account of the subject of incomplete markets, and I would therefore like to recommend it highly.

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