underemployment equilibria

The standard model of general equilibrium is extended by allowing for expectations about supply opportunities by households and firms. In this framework there is typically a 1-dimensional continuum of underemployment equilibria that range from equilibria with arbitrarily pessimistic expectations to equilibria with rather optimistic expectations. An example illustrates the model and highlights some features of underemployment equilibria. The multiplicity of equilibria has a natural interpretation as being the result of coordination failures. The results in this framework are compared with those of the fixprice literature. Extensions to a monetary economy are discussed.

Underemployment of resources refers to the situation where an increase in the resource utilization rate could lead to a Pareto improvement. Typical examples are involuntary unemployment and idle production capacities. There are two quite distinct views on the underemployment of resources. In the standard neoclassical world of the Arrow–Debreu model, underemployment of resources cannot occur. In the competitive equilibrium, involuntary unemployment does not exist, and production capacities are left idle when only such is Pareto optimal.

The Keynesian tradition, in contrast, builds on wage and price rigidities in its explanation of underemployment of resources. Indeed, Keynes’s contribution has been reinterpreted by Clower (1965) and Barro and Grossman (1971) as the economics of general disequilibrium, in which price rigidities lead to quantity constraints for households and firms, which generally have spillover effects in other markets. This lead has been further developed in the fixprice literature, originating in the work of Bénassy (1975), Drèze (1975), and Younès (1975), and in general equilibrium theories on temporary equilibrium (see Grandmont, 1977, for a survey).

Although the fixprice literature stresses wage and price rigidities, Keynes himself postulates that it is possible to encounter self-justifying expectations, beliefs which are individually rational but which may lead to socially irrational outcomes (Keynes, 1936, ch. 12). We therefore like to address the question whether underemployment of resources is possible when expectations are rational, agents optimize, and trade takes place at competitive prices. The underlying reason for underutilization of resources comes from coordination failures, self-justifying expectations which are individually rational but socially suboptimal.

In the literature, one may distinguish three classes of models with coordination failures. The first class consists of rather abstract game-theoretic models following the seminal work of Bryant (1983); see Cooper, 1999, for a state-of-the art account. An important message coming from this stream of the literature is that coordination failures may occur when there are strategic complementarities and positive externalities. However, these models typically lack a coordinating role for the price mechanism.

Strategic models with a coordinating role for prices constitute the second class. Roberts (1987) presents a model that meets these criteria. It is a simple model of a closed economy that allows for coordination failures in a strategic setting. However, outcomes in the second class of models are often not robust to slightly different specifications of the model (Jones and Manuelli, 1992).

The third class of models consists of general equilibrium models of coordination failures. We refer to Citanna et al. (2001) for the most general
presentation of these ideas. The third class leads to results that are robust and general. The methodological assumptions are shared with those of the neoclassical model: agent optimization, market clearing, and rational expectations. Underemployment of resources occurs as a consequence of self-confirming, pessimistic expectations about supply opportunities.

**Competitive equilibria**

Consider the classical general equilibrium model with $H$ households, $F$ firms and $L$ commodities as described in Debreu (1959). A household $h$ is characterized by its consumption set, for the sake of simplicity equal to $\mathbb{R}_+^L$, a utility function $u^h : \mathbb{R}_+^L \rightarrow \mathbb{R}$, and initial endowments $e^h \in \mathbb{R}_+^L$. The feasible production plans of firm $f$ are described by the production possibility set $Y^f$. If firm $f$ chooses production plan $y^f$ and the prices at which trade takes place are $p \in \mathbb{R}^L$, then the firm’s profits equal $p \cdot y^f$. Household $h$ receives a share $\theta^h$ of the profits of firm $f$.

We assume both households and firms to be price takers. If trade takes place at prices $p$, then firm $f$ faces the following profit maximization problem:

$$\max_{y^f \in Y^f} p \cdot y^f.$$ 

Under standard assumptions the firm’s profit maximization problem is well-defined. For the remainder we assume the profit maximizing production bundle to be unique. This can also be shown to hold under standard assumptions, mainly requiring the strong assumption of decreasing returns to scale.

The utility maximization problem of household $h$ reads as follows:

$$\max_{x^h \in \mathbb{R}_+^L} u^h(x^h) \text{ s.t. } p \cdot x^h \leq w^h,$$

where $u^h$ equals the value of the household $h$’s initial endowments, $p \cdot e^h$, plus the household’s share in the firms’ profits, $\sum_f \theta^h p \cdot y^f$, with $y^f$ the profit maximizing production bundle chosen by firm $f$. Under standard assumptions, the maximization problem of the household has a unique solution $x^{*h}$.

Under the usual microeconomic methodological premises of agent optimization and market clearing, together with rational expectations, one defines a **competitive equilibrium** as a price system $p^*$ and an allocation $(x^{*h}, y^{*f}) = (x^{*1}, \ldots, x^{*H}, y^{*1}, \ldots, y^{*F})$ such that at prices $p^*$ households maximize utility by choosing the consumption bundle $x^{*h}$ and firms maximize profits by choosing the production plan $y^{*f}$.

**Rationing**

The puzzle as to how competitive equilibria are achieved in real-world economies remains substantial. First, it is well-known that price adjustment processes need not converge to an equilibrium (Debreu, 1974; Saari and Simon, 1978; Saari, 1985). Blad (1978) stresses that convergence, even if it takes place, can take quite some time. Second, in many situations some agents, or coalitions of agents, set prices at levels not compatible with competitive equilibrium. Drèze (1989) models unions that set wages above competitive levels. Herings (1997) and Tuinstra (2000) show that political interference in the market mechanism can be rational from a partisan point of view and might be responsible for sustained deviations from prices that
clear markets. Third, Drèze and Gollier (1993), Drèze (1997), and Herings and Polemarchakis (2005) argue that certain price rigidities are a welfare-improving response to market incompleteness. This argument is particularly valid for the two forms of underemployment most frequently encountered, namely, unemployed labour and excess capacities, two examples of commodities for which future markets are hardly developed.

For the moment we maintain the assumption that trade takes place at given prices \( p \). Here, \( p \) may or may not be competitive. We are not focusing on a specific theory of non-market clearing prices, but rather are interested in knowing how agents make decisions given that trade takes place at prices \( p \). We deviate from the standard framework and assume that it is not common knowledge whether these prices are competitive or not. Even when all agents know whether prices are competitive or not, it is not necessarily the case that all agents know that all agents know whether prices are competitive or not, and it is even less likely that all agents know that all other agents know that all other agents know whether prices are competitive or not, and so on. Common knowledge of whether prices are competitive requires structural knowledge about the economy, very much at odds with the standard general equilibrium paradigm whereby in a decentralized economy agents only have to maximize utility given the prices that are quoted in the marketplace.

Our price system \( p \) may or may not be competitive. Since this fact is not common knowledge, it no longer makes sense for households and firms to express their unconstrained demands and supplies, and they should form expectations about supply and demand opportunities. One possible choice for these expectations is optimistic expectations: all households and firms do not expect to be constrained in either supply or demand. When prices are competitive, we are back in the situation of competitive equilibrium. The question is: are these the only possible expectations compatible with the microeconomic methodological premises of agent optimization and market clearing, together with rational expectations?

Motivated by the empirical regularity that constraints on the supply side are more common than those on the demand side, as is suggested by unemployment in labour markets or unused capacities in production processes, we follow van der Laan (1980) and Kurz (1982) and restrict attention to constraints on the supply side. Moreover, for the sake of simplicity, we consider point expectations about supply opportunities.

If trade takes place at prices \( p \) and firm \( f \) expects supply opportunities of at most \( \tilde{y}^f \in \mathbb{R}^L \), then firm \( f \)'s profit maximization problem becomes:

\[
\max_{y^f \in Y^f} p \cdot y^f \quad \text{s.t.} \quad y^f \leq \tilde{y}^f.
\]

At this point it should be noted that, if a firm \( f \) does not produce a particular commodity \( l \), the value of \( \tilde{y}^f_l \) is entirely inconsequential. Again, under standard assumptions the firm’s profit maximization problem is well-defined. In fact, the constraints related to the expected supply opportunities ensure that the firm’s profits are bounded from above, a property that does not hold in general for the competitive model. At prices \( p \) and expected supply opportunities of \( \tilde{y}^f \), the supply of firm \( f \), that is, the profit maximizing production plan of firm \( f \), is denoted by \( s^f(p, \tilde{y}^f) \) and the firm’s profits by \( \pi^f(p, \tilde{y}^f) \). The wealth of household \( h \) is then equal to 

\[
w^h = p \cdot e^h + \sum_l \theta_l^h \pi^f(p, \tilde{y}^f).
\]

The utility maximization problem of household \( h \) that trades at prices \( p \), expects supply opportunities equal to \( z^h \), and has budget \( w^h \) equals:
Under standard assumptions, the maximization problem of the household has a unique solution \( d^h(p, x^h, w^h) \).

Since supply may not equal demand, one needs a rule to address discrepancies, called a rationing mechanism. Expected supply opportunities should be related to the rationing mechanism, which determines the allocation in case of excess supplies. For labour markets, one can think of a priority system that determines which worker is the first to become unemployed, who is next, and so on and so forth. Another rationing mechanism would share the burden of unemployment equally among workers, for instance by the imposition of an upper bound on the number of hours worked per week.

For notational convenience we assume the latter rationing mechanism in all markets, implying that in equilibrium rational agents face the same expected supply opportunities, \( \bar{y}^1 = \cdots = \bar{y}^d = -\bar{z}^1 = \cdots = -\bar{z}^h \). We denote the commonly expected supply opportunities by \( r \in \mathbb{R}^l_+ \). At expected supply opportunities \( r \), every firm \( f \) faces the constraint \( y^f \leq r \) and every household \( h \) optimizes utility subject to \(-r \leq x^h - e^h \). All the results remain true with appropriate modifications for general rationing systems; see Herings (1996b) for a survey of rationing systems encountered in the literature.

A firm or household is said to be rationed in the market of commodity \( f \) if the expected supply opportunities in this market are binding. More precisely, a firm \( f \) is rationed in the market of commodity \( f \) at prices \( p \) and expected supply opportunities \( r \) if \( \pi^f(p, \bar{r}) > \pi^f(p, r) \), where \( \bar{r} = +\infty \) and, for \( f \neq f' \), \( \bar{r}_f = r_f \). A household \( h \) is rationed in the market of commodity \( f \) at prices \( p \) and expected supply opportunities \( r \) if \( u^h(d^h(p, -r, w^h)) < u^h(d^h(p, -\bar{r}, w^h)) \), where \( \bar{r} \) is related to \( r \) as before. There is rationing on the market of commodity \( f \) if at least one firm or at least one household is rationed on the market of commodity \( f \).

**Definition.** An underemployment equilibrium of the economy \( \mathcal{E} = ((u^h, e^h)_h, ((Y^f, (\theta^f)_f))_f) \) at prices \( p \) are expected supply opportunities \( r^* \) and an allocation \((x^*, y^*) \) such that

(a) for every firm \( f \), \( y^f = s^f(p, r^*) \),
(b) for every household \( h \), \( x^h = d^h(p, -r^*, w^h) \), where \( w^h = p \cdot e^h + \sum_f \theta^f \pi^f(p, r^*) \),
(c) \( \sum_h x^h = \sum_h e^h + \sum_f y^f \).

**An example.**

As a simple example, let us consider an economy with one household, one firm, and two commodities. Let us interpret the commodities as leisure and an aggregate consumption good, and suppose that the household owns initially one unit of leisure and nothing of the consumption good, \( e = (1, 0) \). The household’s utility function is Cobb-Douglas, \( u(x) = x_1 x_2 \). The firm transforms the labour input into output by the production function \( y_2 = \sqrt{2y_1 - 3y_2} \), where \( y_1 \leq 0 \). When we normalize the price of output to be 1, turning the wage rate into the real wage rate, it is not hard to verify that the competitive equilibrium is given by \( p^* = (1, 1) \), \( x^* = (\bar{z}, \bar{z}) \), and \( y^* = (\bar{z}, \bar{z}) \).

Now suppose that it is possible to trade at the competitive equilibrium prices, so \( p = (1, 1) \), but it is not common knowledge that these prices are...
competitive, and as a consequence firms and households form point expectations on supply opportunities \( r = (r_1, r_2) \). We want to verify whether such expectations can be self-confirming. It is easily verified that for each \( r^* \in [0, \frac{1}{3}] \) there is an underemployment equilibrium with expected supply opportunities \( r^* \) given by \( r_1^* = \frac{3 \sqrt{3} r^*}{3 r^*}, x^* = (1 - r_1^*, r_2^*), \) and \( y^* = (-r_1^*, r_2^*) \). The household expects a constraint on labor supply equal to \( -r_1^* \) yielding labor income \( r_1^* \). The firm expects a constraint on the supply of output equal to \( r_2^* = \frac{2}{3} \sqrt{3} r^*_2 \). It optimally demands an amount of labor equal to \( r_1^* \), leading to profits \( \frac{2}{3} \sqrt{3} r^*_2 - r_1^* \). Notice that the optimal labor demand by the firm equals the constraint on labor supply anticipated by the household. The household’s capital income is equal to \( \frac{2}{3} \sqrt{3} r^*_2 - r_1^* \), leading to total income of \( \frac{2}{3} \sqrt{3} r^*_2 \), to be spent on the aggregate consumption good. The optimal demand of the household for the aggregate consumption good equals the supply opportunities expected by the firm, thereby confirming those expectations. There is rationing in both markets. The household is rationed in the labour market and the firm in its output market.

Finally, every \((r_1^*, r_2^*)\) with \( r_1^* \geq \frac{1}{3} \) and \( r_2^* \geq \frac{2}{3} \) sustains an underemployment equilibrium that coincides with a competitive equilibrium in terms of the allocation reached, \( x^* = \left( \frac{2}{3}, \frac{1}{3} \right), y^* = (-1, \frac{2}{3}) \). In this case, there is no market with rationing.

In the example, two extreme underemployment equilibria stand out. One is the underemployment equilibrium with completely pessimistic expectations about supply opportunities, \( r^* = (0, 0), x^* = (1, 0), y^* = (0, 0) \). The other is the underemployment equilibrium with expectations about supply opportunities that are sufficiently optimistic to obtain the competitive allocation; the minimally optimistic expectations to achieve this are \( r^* = \left( \frac{1}{3}, \frac{2}{3} \right) \). These extreme underemployment equilibria are connected by a set of underemployment equilibria with more moderate expectations on supply opportunities.

In the example, trade was supposed to take place at competitive prices to highlight underemployment caused by mis-coordination of expectations and not by relative prices that are incompatible with competitive equilibrium. One may argue that it is a probability zero event that trade takes place at competitive prices. The crucial features of the example remain unchanged when trade takes place at non-competitive prices. Suppose that trade takes place at a real wage \( p_1 \) above the competitive wage rate of 1. It can be verified that there is still a no-trade equilibrium sustained by completely pessimistic expectations on expected supply opportunities. Although the competitive allocation is no longer feasible when the real wage is above the competitive level, it can be verified that there is also an underemployment equilibrium where the firm does not face rationing and the household observes rationing of its labour supply; the minimally optimistic expectations on supply opportunities that sustain this equilibrium are given by

\[
 r^* = \left( \frac{1}{3(p_1)^2}, \frac{2}{3p_1} \right)
\]

leading to consumption and production

\[
x^* = \left( 1 - \frac{1}{3(p_1)^2}, \frac{2}{3p_1} \right), \quad y^* = \left( -\frac{1}{3(p_1)^2}, \frac{2}{3p_1} \right).
\]

The same underemployment allocation is sustained by more optimistic expectations

\[
r_1^* = \frac{1}{3(p_1)^2} \quad \text{and} \quad r_2^* \geq \frac{2}{3p_1}.
\]
Again, the two extreme equilibria are connected by a continuum of underemployment equilibria, with expectations ranging from completely pessimistic to expectations that sustain an underemployment equilibrium without rationing of the firm and with rationing of the labour supply of the household.

When the real wage $p_1$ is below the competitive level, there is still an underemployment equilibrium with completely pessimistic expectations about supply opportunities and no trade. There is also an underemployment equilibrium without rationing of the household but with rationing of the firm’s supply of output. Let $\bar{r}_2$ be equal to $4(\sqrt{1 + 3(p_1)^2})/6p_1$. Notice that $\bar{r}_2$ is below $2/3$ when the real wage $p_1$ is below 1. The minimally optimistic expectations that sustain an equilibrium without rationing of the household are given by $r_n^* = (1 - \bar{r}_2/p_1, \bar{r}_2)$ leading to an allocation $x_n^* = (\bar{r}_2/p_1, \bar{r}_2^2)$, $y_n^* = (\bar{r}_2/p_1 - 1, \bar{r}_2)$. The same underemployment equilibrium allocation is sustained by the more optimistic expectations $r_n^* = (1 - \bar{r}_2/p_1, \bar{r}_2^2)$ and $r_n^* = \bar{r}_2^2$. The two extreme underemployment equilibria are connected by a continuum of underemployment equilibria with more moderate expectations on supply opportunities.

Animal spirits

The example suggests that in general there is a 1-dimensional continuum of equilibria, ranging from a no-trade equilibrium with completely pessimistic expectations to an equilibrium with rather optimistic expectations and without rationing in at least one market. This result is almost true, except that the case with completely pessimistic supply expectations leads to zero income for the households, a case that is well-known to be problematic for equilibrium existence. Inspired by preliminary results in van der Laan (1982), Herings (1996a; 1998) and Citanna et al. (2001) provide conditions under which the following result holds.

**Theorem.** Under standard assumptions, the economy $E = ((u^h, e^h)_h, (Y^f, (0)^f)_h)$ where trade takes place at prices $p$ possesses a connected set of underemployment equilibria $E$ such that for every $\rho \in (0, \to)$, there is an equilibrium $(r^*, x^*, y^*)$ in $E$ with $\max_i r_i^* = \rho$. (A set is path-connected if any two points in the set can be connected by a path that does not leave the set. Path-connectedness implies connectedness, a slightly weaker topological property of sets, which loosely speaking means that the set consists of one piece.)

The theorem gives general equilibrium underpinnings to the Keynesian ideas that changes in expectations, also referred to as animal spirits, can affect equilibrium economic activity, in particular the level of output and employment. The theorem rules out the case with completely pessimistic expectations, corresponding to $\max_i r_i^* = 0$. It shows that the set $E$ links equilibria with arbitrarily pessimistic expectations ($\max_i r_i^*$ arbitrarily small, but positive) to equilibria with rather optimistic expectations ($\max_i r_i^*$ arbitrarily large). Notice that the condition $\max_i r_i^*$ arbitrarily large only implies that for one market expectations are sufficiently optimistic to rule out rationing. The expectations on supply opportunities on other markets might still be completely pessimistic.

In the absence of production, the statement of the theorem can be simplified somewhat. It is shown in Herings (1998), under standard assumptions, that the economy $E = ((u^h, e^h)_h)$ where trade takes place at prices $p$ possesses
a connected set of underemployment equilibria \( E \) such that for every \( \rho \in [0, \to) \) there is an equilibrium \((\rho^*, x^*, y^*)\) in \( E \) with \( \max_i r_i^* = \rho \). In exchange economies, the connected set includes an underemployment equilibrium with completely pessimistic expectations.

The theorem demonstrates that the set of equilibria is at least 1-dimensional. In general, one should expect the dimension of the set of equilibria to be exactly equal to 1. The reason is that the model postulates \( L \) free variables corresponding to the expected supply opportunities \( r \) and \( L \) market clearing conditions. Let \( E \) be an economy where trade takes place at prices \( p \) and let \( z : R^L_+ \to \mathbb{R}^L \) denote the excess demand function of the economy, a function of expected supply opportunities \( r \). Because of Walras's law, it holds that for every \( r \in R^L_+ \), \( p \cdot z(r) = 0 \). This implies that there are only \( L - 1 \) independent market clearing conditions. Since there are \( L \) free variables, this leaves a 1-dimensional solution set.

At this point it should be observed that the same reasoning also applies to the standard competitive model. And indeed, in general the set of competitive equilibria is 1-dimensional too. Whenever \((\rho^*, x^*, y^*)\) constitutes a competitive equilibrium, so does \((\lambda \rho^*, x^*, y^*)\) for \( \lambda \) positive. However, in the standard competitive model, the excess demand function is homogeneous of degree 1, implying that the same allocation is sustained by \( \rho^* \) and \( \lambda \rho^* \). The homogeneity property holds for prices but not for expectations. In general, the excess demand function \( z \) is not homogeneous of any degree, and it is not the case that \( \lambda \rho^* \) is part of an underemployment equilibrium when \( \rho^* \) is.

Generically, the set \( E \) of the theorem contains a 1-dimensional set of distinct equilibrium allocations.

**Coordination failures**

The theorem makes clear that a multiplicity of equilibria results even when prices are competitive. The interpretation of the multiplicity of equilibria as coordination failures and the link to the macroeconomic literature on coordination failures were made in Drezé (1997). It would be tempting to conclude that, when trade takes place at competitive prices, then the connected set of underemployment equilibria contains the competitive equilibrium allocation. Although it is true that the competitive equilibrium allocation is an underemployment equilibrium allocation sustained by trade at competitive prices coupled with sufficiently optimistic expectations, it is possible to produce examples where it is outside the connected set of the theorem (Citanna et al., 2001).

Under an additional assumption akin to gross substitutability, the following result holds. If trade takes place at competitive equilibrium prices \( p \), then the economy \( \delta = ((u^b, e^h)_b, (Y^f, (0, \beta)_h)_f) \) possesses a connected set of underemployment equilibria \( E \) such that for every \( \rho \in (0, \to) \) there is an equilibrium \((\rho^*, x^*, y^*)\) in \( E \) with \( \max_i r_i^* = \rho \), and for every \( \rho \in (0, \to) \) there is an equilibrium \((\rho^*, x^*, y^*)\) in \( E \) with \( \min_i r_i^* = \rho \). More precisely, the following assumption on the aggregate excess demand function suffices. If \( r, \tilde{r} \in \mathbb{R}^L_+ \) with \( r \leq \tilde{r} \) and \( r_f = \tilde{r}_f \), then \( z_f(r) \leq z_f(\tilde{r}) \). The interpretation of the assumption is the following. A weak increase in expected supply opportunities in markets different from \( f \) should lead to a weak increase in the excess demand for commodity \( f \).

This assumption, though strong, is not unreasonable. On the household side, a household may lower its supply of commodity \( f \) in exchange for more supply of another commodity, for instance if the household switches to a more attractive job. A household may also increase its demand for com-
modity \ell as a consequence of higher income. Indeed, more expected supply opportunities of commodities different from \ell weakly increase the household’s income, which will lead to more demand of commodity \ell if it is a normal good. On the producer side, if \ell is an output for some firm, then increased supply opportunities of other goods, lead to a weakly lower supply of commodity \ell, when the firm directs more inputs to the production of the other goods. If \ell is an input for a firm, then increased supply opportunities of other goods naturally lead to more production, and thereby an increased input demand, in particular for commodity \ell. Notice that the assumption needs to hold only in the aggregate.

Efficiency

It is not hard to argue that underemployment equilibria are not Pareto optimal in general. As soon as there are two commodities, \ell and \ell', and two households \( h \) and \( \hat{h} \), such that household \( \hat{h} \) is rationed in the market for commodity \ell but not in the market for commodity \ell, whereas household \( h \) is not rationed in the market for commodity \ell', then it follows almost immediately that the marginal rate of substitution of commodity \ell for \ell' is not the same for households \( h \) and \( \hat{h} \). This contradicts Pareto optimality.

It has been argued before that price rigidities may emerge for efficiency reasons. The argument of the previous paragraph makes clear that such is not the case in a complete markets setting when coordination failures are absent. In an incomplete market setting, however, Drèze and Gollier (1993) and Herings and Polemarchakis (2005) show that price rigidities may lead to equilibria that are Pareto superior to competitive equilibria. In general, it will depend on the magnitude of the coordination failures, whether or not welfare improvements result.

Extensions

For illustrative purposes, we have so far considered the simplest case where trade takes place at predetermined prices for all commodities. It is not hard to generalize this set-up substantially, and allow for general lower bounds \( p \) and upper bounds \( \hat{p} \) that define the set of admissible prices at which trade may take place. The notion of underemployment equilibrium should then be extended by the requirement that only when the price of a commodity \ell equals its lower bound is rationing of the supply of commodity \ell allowed for. In such a more general setting, it may be interesting to also allow for demand rationing when the price of a commodity equals its upper bound. Allowing for demand rationing will enlarge the set of equilibria. By taking all \( p_\ell \) equal to \(-\infty\) and all \( \hat{p}_\ell \) to \( \infty \), we obtain the notion of competitive equilibrium as a special case. The results we mentioned before remain true in this more general set-up.

The existence of a continuum of underemployment equilibria is a robust phenomenon. This seems to be in striking contrast with the conclusion of the fixprice literature, where equilibria are typically locally unique. The reason for this apparent disparity is that the fixprice literature puts one additional constraint on the equilibrium set. It is assumed that there is no rationing in the market of an a priori determined numeraire commodity, called money. Not only is the interpretation of one of the commodities as money controversial, it is also misleading as far as the indeterminacy of equilibrium is concerned.
Suppose we follow Drèze and Polemarchakis (2001) and extend the model by a model of a central bank, which produces money at no cost. Households and firms need money to make transactions, a particular example being a cash-in-advance transactions technology. The central bank sets the nominal interest rate at which it accommodates all money demand. The central bank redistributes the revenues from seigniorage to the households in the form of dividends. Money does not enter into the utility function of the households.

This model fits exactly into the framework discussed so far. Without loss of generality, commodity $L$ may serve as money. The central bank can be modelled as firm $F$, which can produce the output money in arbitrary amounts without using inputs. The profits of firm $F$ are equal to the seigniorage of the central bank. The price of money is equal to the interest rate. Since the central bank accommodates money demand, the money supply of the central bank is equal to that of a profit-maximizing firm $F$ that expects a constraint on the supply of commodity $L$ equal to the aggregate demand for commodity $L$ (for strictly positive interest rates). Our theorem on the existence of a connected set of underemployment equilibria therefore applies to the case where money is explicitly introduced, thereby contradicting the determinacy results obtained in the fixprice literature.

We have shown how underemployment equilibria result in a general equilibrium model where agents are allowed to form expectations on expected supply opportunities. We analyse whether such expectations can be self-confirming and argue that, even at competitive prices, a continuum of equilibria results, including an equilibrium with approximately no trade and a competitive equilibrium. Such equilibria also arise at other price systems, but are then a consequence of both self-confirming pessimistic expectations and of prices incompatible with competitive equilibrium. Expected supply opportunities in underemployment equilibria bear a close resemblance to the self-justifying expectations of Keynes (1936), beliefs which are individually rational but socially suboptimal. The further study of underemployment equilibria in models with time and uncertainty, incomplete markets, price-setting agents, and a monetary authority features prominently on the research agenda, as it would allow for explicit links with the modern macroeconomic literature on inflation, output and unemployment.

P. Jean-Jacques Herings

See also

- determinacy and indeterminacy of equilibria;
- existence of general equilibrium;
- fixprice models;
- general equilibrium (recent developments);
- involuntary unemployment;
- money and general equilibrium;
- rationing.

Bibliography


**Index terms**

agent optimization
animal spirits
Arrow–Debreu model of general equilibrium
Cobb–Douglas functions
competitive equilibrium
coordination failures
economics of general disequilibrium
excess capacity
excess demand
fixprice models
game theory
general equilibrium
general equilibrium models of coordination failures
incomplete markets
involuntary unemployment
Keynes, J.M.
Keynesianism
market clearing
neoclassical model
non-market clearing prices
Pareto optimality
path connectedness
positive externalities
price rigidities
rational expectations
rationing
seigniorage
self-justifying expectations
spillover effects
strategic complementarities
supply opportunities
temporary equilibrium
underemployment equilibria
wage rigidities
Walras’s law

Index terms not found:

Arrow–Debreu model of general equilibrium
Cobb–Douglas functions
excess capacity
fixprice models
game theory
general equilibrium models of coordination failures
Keynesianism
non-market clearing prices
wage rigidities