Either or Both Competition:
A "Two-sided" Theory of Advertising with Overlapping Viewerships

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Abstract

This paper develops a new model of platform competition in media markets, allowing viewers to use multiple platforms. Platforms do not steal viewers from each other, instead they affect the viewer composition and thereby the relative value of marginal and inframarginal viewers on the other platform. We label this form of competition “either or both.” We find that platform ownership does not affect advertising levels, despite nontrivial strategic interaction between platforms. This result holds for general demand functions and advertising technologies. We show that the equilibrium advertising level is inefficiently high. We also demonstrate that entry of a platform leads to an increase in the advertising level if viewer preferences for the platforms are negatively correlated. This is in contrast with predictions of standard models with “either/or” competition, but it is consistent with data from the U.S. cable TV industry.

Keywords: Platform Competition, Two-Sided Markets, Market Entry, Multi-Homing, Viewer Preference Correlation.

JEL-Classification: D43, L13, L82, M37

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1 Introduction

This paper studies the incentives to provide advertising opportunities in markets served by competing media platforms. Online advertising networks, such as the Google and Yahoo ad-networks, and traditional broadcasting stations, such as CNN and Fox News, are among the most prominent examples. In these markets platforms fight for viewer attention and in particular for the accompanying stream of advertising revenues. A central question in the ongoing debate about the changing media landscape is how competition affects advertising levels and revenues.\(^1\)

The traditional approach in media economics posits that viewers have idiosyncratic tastes about media platforms, and stick to those they like best.\(^2\) So, if anything, viewers choose either one platform or some other. Competition is for exclusive viewers as all platforms are restricted a priori to be (imperfect) substitutes. A common argument regarding the impact of competition in this framework goes as follows. The amount of advertising supplied by platforms can be considered the shadow price that viewers pay to satisfy their content needs. Competition typically lowers prices as platforms try to woo viewers from their rivals. So, as the argument goes, increased competition ought to reduce advertising levels.

While compelling, this line of reasoning recently came under attack for three main reasons. First, it fails to account for the fact that many viewers satisfy their content needs on multiple platforms. For example, many football fans watch multiple games during the course of a week, which are often broadcasted on different channels. This is increasingly so as content moves from paper and TV towards the Internet. In fact, many contend that a distinguishing feature of “online consumption” is the users’ increased tendency to spread their attention across a wide array of outlets. Table 1 shows the reach of the six largest online advertising networks, that is, the fraction of the U.S. Internet users who, over the course of December 2012, visited a website belonging to a given network. This table shows that while Google can potentially deliver an advertising message to 93.9% of all Internet users, the smallest of the six networks (run by Yahoo!), can deliver a whopping 83.3%\(^3\). The table highlights a key feature of these markets whose implications for market outcomes are left unexplored: different platforms provide advertisers alternate means of reaching same users.

\begin{table}[h]
\centering
\begin{tabular}{|c|c|c|c|}
\hline
Rank & Property & Unique Visitors (000) & % Reach \\
\hline
1 & Google Ad Network\(^*\)* & 208,074 & 93.9 \\
2 & Specific Media\(^*\)* & 198,119 & 89.5 \\
3 & Federated Media Publisher Network\(^*\)* & 193,453 & 87.3 \\
4 & AOL Advertising\(^*\)* & 186,595 & 84.2 \\
5 & AT&T AdWorks\(^*\)* & 185,757 & 83.9 \\
6 & Genome from Yahoo\(^*\)* & 184,526 & 83.3 \\
\hline
\end{tabular}
\caption{Top 20 Ad Networks/Buy Side Networks}
\end{table}

A second concern with the traditional approach is that it is inconsistent with evidence that advertising

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\(^1\)The issue is whether the wider array of choices and media formats can potentially upend the traditional cross-subsidization business model in which advertising pays the bills. For example, see the Federal Trade Commission’s report, ‘The Information Needs of Communities’ (2011).

\(^2\)For example, see Anderson and Coate (2005) and several follow-up papers. We provide a detailed literature review in the next section.

levels often rise when competition increases. For example, the wave of channel entry during the 1990s in the US cable TV industry seemed to coincide with an increase in advertising levels on many channels (this is usually referred to as the Fox News puzzle). In the Appendix, using a dataset provided by Kagan-SNL, we show that an increase in the number of US cable TV channels market is on average indeed associated with an increase in advertising levels on incumbent channels.4

A third concern with the above argument is that it does not match the public policy debate around advertising bans or caps, e.g. the debate surrounding France’s 2011 ban on prime-time advertising on public TV. Private broadcasters typically oppose lifting such advertising bans. This opposition is hard to rationalize with the above argument as public broadcasters should be perceived as tough competitors for viewers precisely because they do not carry annoying ads.

Motivated by these concerns we develop a theory of market provision of informative advertising that allows viewers to consume content on multiple platforms. So with two platforms, viewers can choose either one platform or both (or none). Specifically, we work under the (extreme) assumption that viewer demand for one platform does not affect the demand for another platform. This is what we call either or both competition in contrast with the either/or framework discussed above.

We claim that either or both competition is an appealing alternative to existing models for multiple reasons. First, it is a good approximation of reality in some non-trivial contexts, e.g., if choosing Facebook for online social social networking is orthogonal to choosing Yelp for online restaurant reviews.5 Second, even when neither assumption is a good approximation, the analysis provides a natural theoretical benchmark to cover the opposite end of the spectrum than the traditional either/or models. Moreover, independence of demand across platforms allows generality in other dimensions, including the advertising technology, and the viewer preference correlation across different platforms. This in turn allows us to address questions that are not tractable in the traditional framework, such as how viewer preference correlation affects equilibrium advertising levels, and how it influences the effect of platform entry.6

The baseline model features two platforms, with continuums of viewers and of advertisers. In line with the literature, platforms simultaneously choose the total quantity of ads. These ads are subsequently allocated to advertisers according to a simple contracting environment in which each platform offers a contract specifying a price for a given advertising intensity. We do not impose a specific functional form on either the distribution of viewer preferences or on the advertising technology.

A key equilibrium property of the either/both formulation is that viewers who are not exclusive to a platform (so called “multi-homing” or “overlapping” viewers) are less valuable than exclusive viewers in equilibrium, because advertisers can reach overlapping viewers through multiple platforms. As a result, platforms can only charge the incremental value of advertising to overlapping viewers via an additional platform. By contrast, platforms are effectively monopolists with respect to selling advertising to their

4 Related to this, recorded advertising levels are fairly high even in markets that are widely considered fairly competitive, such as the US Radio broadcasting markets (Jeziorski 2012).

5 According to the source supra cited, Facebook and Yelp are among the most-visited U.S. websites and in fact belong to different advertising networks.

6 Existing models either assume a Hotelling framework, imposing perfect negative correlation in viewer preferences for two platforms, or a representative consumer framework. In contrast, our framework allows for viewer preferences to be correlated in any way between platforms.
exclusive viewers, and can extract the full surplus for these transactions from advertisers.\footnote{That multi-homing viewers are worth less to advertisers is consistent with the well-documented fact that the per-viewer fee of an advertisement on programs with more viewers is larger. In the U.S., e.g., Fisher, McGowan and Evans (1980) find this regularity. ITV, which is the largest TV network in the UK, enjoys a price premium on its commercials, which, despite entry of several competitors, increased steadily in the 1990s. This trend is commonly referred to as the “ITV premium puzzle”. Our model can account for this puzzle since reaching the same number of eyeball pairs through broadcasting a commercial to a large audience implies reaching more viewers than reaching the same number of eyeball pairs through a series of commercials to smaller audiences, because the latter audiences might have some viewers in common. See Ozga (1960) for an early observation of this fact.} This implies that platforms do not only care about the overall viewer demand \textit{level}, as in existing models, but also about its \textit{composition}, i.e., the fraction of exclusive versus overlapping viewers. To stress the importance of composition in media markets, the appendix includes an excerpt from the sales pitch for Google’s Display Network (GDN). It employs proprietary data to assess the effect for an advertising campaign on auto insurance. A “key takeaway” of the sales pitch is that the GDN “exclusively reaches 30% of the auto-insurance seekers” that do not visit Yahoo, 36% that do not visit Youtube and so on.

When a media platform unilaterally increases advertising, its marginal customers switch off. As some of these customers were not exclusive, this increase changes the \textit{composition} of its rival’s customer base, which shifts the rival’s marginal revenue curve. So there is non-trivial strategic interaction. Nevertheless, our first finding is that competition is disabled in the following sense: a monopolist running both platforms provides the same advertising quantity that competing duopolist would provide in equilibrium. The intuition for this “neutrality result” proceeds as follows: since a monopolist can extract more rents from advertisers than competing platforms can, the monopolist has a greater incentive to advertise. However, a competing platform receives a lower rent because it can only charge advertisers a low price for access to overlapping viewers. But this implies that a competing platform is hurt less when increasing its advertising by the overlapping viewers who switch off. Overall, these two effects balance out, leading to the same amount of advertising in both scenarios. We show that this result is surprisingly robust and extends to the cases in which a) platforms can charge prices on the viewer side of the market, b) there is an arbitrary number of platforms, and c) platforms compete by posting screening contracts to privately-informed advertisers.

Next, and somewhat surprisingly, we find that entry could lead to an \textit{increase} in the incumbent’s supply of ads. The neutrality result suggests that this finding does \textit{not} reflect enhanced competition for advertisers due to overlapping viewers. Rather, the intuition relies on understanding how the advertisers’ willingness to pay for access to the incumbent’s customer base changes when they are granted the option to advertise through an \textit{additional} platform. Due to decreasing marginal returns to advertising, such an additional communication channel leads to inefficient extra advertising to multi-homing viewers. We call this the \textit{duplication effect}. Concerns about inefficient duplication, other things being equal, lead to lower advertising levels. However, before entry all viewers are valuable exclusive viewers of the incumbent platform, implying that an increase in advertising induces these viewers to switch off. By contrast, after entry some of the marginal viewers are multi-homers and are therefore less valuable. We call this the \textit{business sharing} effect, which, other things being equal, leads to an increase in advertising quantity. The overall effect is ambiguous. Nonetheless, we provide an intuitive and full characterization of entry in terms of the elasticities of viewer demand and of the properties of the communication technology, that is, in terms of empirical objects.
Turning to the effects of viewer preference correlation, we find that the higher the preference correlation between two platforms, the lower the equilibrium advertising levels. This result follows because, with positively correlated preferences, a relatively large share of viewers are overlapping, leading to a large duplication effect. This implies that negative preference correlation is beneficial from the platforms’ perspective as it allows to considerably increase the reach via higher advertising quantities at little costs in terms of duplication.

Consequently, the more negative the viewer preference correlation between platforms, the more likely it is that entry leads to increased advertising. For example, CNN likely increases its advertising level after entry of FOX News. By contrast, if the preference correlation is positive, as is the case for sports and leisure programs, incumbents decrease advertising upon entry. In the Appendix we show that these predictions are also consistent with data from the US cable TV industry. Even though on average we find that entry leads to an increase in incumbents’ advertising levels, in market segments where viewer preferences are strongly positively correlated, such as the sports segment, incumbents decrease advertising upon entry.8

Our results are important both for economic theory and for policy discussions on changes in the media landscape, e.g. how to evaluate mergers of broadcasting companies. In particular, the neutrality result shows that mergers in media markets can be neutral with respect to social welfare. As a corollary to the neutrality result, we should not expect policies that require spin-offs to mitigate inefficient over-provision of advertising, as the caps and bans on advertising proposed by policy makers seem to suggest. Also competition between channels does not necessarily lead to a fall in the advertising levels and might even increase the inefficient overprovision.

The rest of the paper is organized as follows: Section 2 discusses related literature. Section 3 introduces the model and Section 4 presents the equilibrium analysis. Section 5 performs a welfare analysis. Section 6 considers market entry. Section 7 explores the effects of viewer preference correlation. Sections 8 and 9 consider extensions of the model with viewer pricing, and with heterogeneous advertisers, respectively. Section 10 concludes.

2 Related Literature

The traditional framework in media economics makes the assumption that viewers do not switch between channels, but rather select the program they like most, e.g. Spence and Owen (1977) or Wildman and Owen (1985). These early works usually do not allow for endogenous advertising levels or two-sided externalities between viewers and advertisers.

The seminal paper modelling the television market as a two-sided market with platforms that compete for viewers and for advertisers is Anderson and Coate (2005).9 In their model, viewers are distributed on a Hotelling line with platforms located at the endpoints. Similar to early works, viewers watch only

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8We include these empirical correlations from the US cable TV market in the Appendix only to show that some basic features of the data are consistent with our model, but hard to reconcile with either/or models. Due to data limitations, a more thorough empirical investigation, e.g. to prove causal relationships, is beyond the scope of our paper.

9For different applications of two-sided market models, see e.g., Rochet and Tirole (2003, 2006), Armstrong (2006) and Weyl (2010).
one channel while advertisers can buy commercials on both channels.\footnote{In Section 5 of their paper Anderson and Coate (2005) extend the model by allowing a fraction of viewers to switch between channels, that is, to multi-home.} In this framework, Anderson and Coate (2005) show, among several other results, that the number of entering stations can either be too high or too low compared to the socially optimal number, or that the advertising level can be above or below the efficient level.

Anderson and Coate’s basic model has been extended and modified in several ways. For example, Gabszewicz, Laussel and Sonnac (2004) allow viewers to mix their time between channels, Peitz and Valletti (2008) analyze optimal locations of stations, and Reisinger (2012) considers single-homing of advertisers. Dukes and Gal-Or (2003) explicitly consider product market competition between advertisers and allow for price negotiations between platforms and advertisers, while Choi (2006) and Crampes, Haritchabalet and Jullien (2009) consider the effects of free entry of platforms. Finally, Anderson and Peitz (2012) allow advertising congestion and show that it can also lead to increased advertising rates after entry of new platforms.

These papers do not allow viewers to watch more than one station, i.e., they assume either/or competition, and usually consider a spatial framework for viewer demand.\footnote{A different framework to model competition in media markets is to use a representative viewer who watches more than one program. This approach is developed by Kind, Nilssen and Sørgard (2007) and is used by Godes, Ofek and Savary (2009) and Kind, Nilssen and Sørgard (2009). These papers analyze the efficiency of the market equilibrium with respect to the advertising level and allow for viewer payments. Due to the representative viewer framework, they are not concerned with overlapping viewers or viewer preference correlation.} By contrast, our paper allows for viewers watching multiple channels, and for a much more general viewer demand system.\footnote{For a similar kind of demand structure, which also allows overlapping of consumers, albeit in a very different context, see Armstrong (2013).} In addition, we allow for a general advertising technology.

The paper that is closest to ours is Anderson, Foros and Kind (2012b), who also consider the case of multi-homing viewers and, in addition, allow for endogenous platform quality.\footnote{See also Anderson, Foros, Kind and Peitz (2012).} They show that with multi-homing viewers, advertising levels increase after entry and generate different equilibrium configurations in which either one or both sides multi-home. However, the modelling structure is very different from ours. For example, they consider an adapted Hotelling framework developed by Anderson, Foros and Kind (2012a) to focus on quality choice, suppose that the value of overlapping viewers equals zero, and consider linear pricing to advertisers by platforms. By contrast, we fix quality, but allow for general viewer demand functions, advertising technology, and payments.

Two other papers that allow for multi-homing viewers are Athey, Calvano and Gans (2012) and Bergemann and Bonatti (2011, in Sections 5 and 6). In the former model, the effectiveness of advertising can differ between users who switch between platforms and those who stick to one platform because of imperfect tracking of users. The latter paper explicitly analyzes the interplay between perfect advertising message targeting in online media markets and imperfect targeting in traditional media. In contrast to our model, the two papers above are mainly concerned with different tracking/targeting technologies and do not allow for advertisements generating (negative) externalities on viewers, which is at the core of our model.
3 The Model

The model features a continuum of heterogeneous viewers, a continuum of homogeneous advertisers and two platforms indexed by $i \in \{1, 2\}$.

**Viewer demand system and independence**

Viewers are parametrized by their reservation utility $(q_1, q_2) \in \mathbb{R}^2$ for platforms 1 and 2, where $(q_1, q_2)$ is distributed on some subset of $\mathbb{R}^2$ according to a bivariate probability distribution with smooth joint density denoted $h(q_1, q_2)$. A viewer of $(q_1, q_2)$-type joins platform $i$ if and only if $q_i - \gamma n_i \geq 0$, where $n_i$ is the amount of ads on platform $i$ and $\gamma > 0$ is a nuisance parameter. Given the amount of advertising on each platform, we can back out the demand system:

- **Multi-homers**: $D_{12} := \text{Prob}\{q_1 - \gamma n_1 \geq 0; q_2 - \gamma n_2 \geq 0\}$,
- **Single-homers 1**: $D_1 := \text{Prob}\{q_1 - \gamma n_1 \geq 0; q_2 - \gamma n_2 < 0\}$,
- **Single-homers 2**: $D_2 := \text{Prob}\{q_1 - \gamma n_1 < 0; q_2 - \gamma n_2 \geq 0\}$,
- **Zero-homers**: $D_0 := 1 - D_1 - D_2 - D_{12}$.

To ensure uniqueness of the equilibrium and interior solutions, we need to assume that the demand functions are well-behaved. Ultimately, this boils down to assumptions on the joint distribution function $H(q_1, q_2)$. However, it is not necessary to spell out assumptions on this function, since we will later work with the demand functions. Hence, we make the assumptions directly on the demand functions. In particular, we assume that for each $i = 1, 2$ and $j = 3 - i$,

$$\frac{\partial^2 D_i}{\partial^2 n_i} \leq 0, \quad \frac{\partial^2 D_{12}}{\partial^2 n_i^2} \leq 0 \quad \text{and} \quad \left| \frac{\partial^2 D_i}{\partial n_i \partial n_j} \right| \geq \left| \frac{\partial^2 D_i}{\partial n_i^2} \right|.$$

These regularity assumptions are stricter than necessary. If instead each of the three inequalities were violated but only slightly so, we still have interior solutions. For a detailed discussion of why the above conditions ensure concavity of the profit function and uniqueness of the equilibrium, see e.g., Vives (2000).

**Timing and platforms’ choices**

The platforms compete for viewers and for advertisers. In the basic model, the platforms receive payments only from advertisers but not from viewers. The timing of the game is as follows. At stage 1, the platforms simultaneously set the total advertising capacities $n_1$ and $n_2$. At stage two, viewers observe $n_1$ and $n_2$ and choose which platform to join, if any. At stage three, platforms simultaneously announce menus of contracts. A contract offered by platform $i$ is a pair $(t_i, m_i) \in \mathbb{R}^2_+$, which specifies an advertising intensity $m_i \geq 0$ in exchange for a monetary transfer $t_i \geq 0$. Finally, at stage four, advertisers simultaneously decide which contract(s), if any, to accept at each platform. Below we will show that in our basic model with homogeneous advertisers, each platform only offers one contract in equilibrium, and it is accepted by all advertisers.\(^{15}\)

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\(^{14}\)We consider viewer pricing in Section 8.

\(^{15}\)In Section 9 we consider a model with heterogeneous advertisers in which platforms do want to offer multiple contracts in equilibrium.
To make sure that the announced advertising levels are consistent with realized levels after stage four, we assume that if total advertising intensities accepted by the advertisers at platform \( i \) exceed \( n_i \) then platform \( i \) obtains a large negative payoff.\(^{16}\) Therefore, our game is similar to Kreps and Scheinkman (1983), i.e., in the first stage platforms choose an advertising capacity that puts an upper bound on the amount of advertising slots they can sell subsequently.

The solution concept we use throughout the paper is subgame perfect Nash equilibrium (SPNE).

**Advertising technology**

There is a mass 1 of advertisers. Advertising in our model is informative. Let \( \omega \geq 0 \) denote the expected return of informing a viewer about a product. In line with the literature, e.g., Anderson and Coate (2005) or Crampes, Haritchabalet and Jullien (2009), we assume that viewers are fully expropriated of the value of being informed.\(^{17}\)

The mass of informed viewers is determined by the intensity of advertising the particular advertiser purchased. We denote the probability with which a single-homing viewer on platform \( i \) becomes informed of an advertiser’s good by \( \phi_i(m_i) \). We assume that \( \phi_i \) is smooth, nondecreasing and strictly concave, with \( \phi_i(0) = 0 \). That is, there are positive but diminishing returns to advertising. Likewise, the probability that a multi-homing viewer becomes informed about the product of an advertiser who puts ads on both platforms depends on the number of ads, \( m_1 \) and \( m_2 \), that the viewer is exposed to. We assume \( \phi_{12}(m_1, m_2) \) is smooth with \( \partial \phi_{12}/\partial m_i \geq 0 \) and \( \phi_{12}(m_i, m_j) = \phi_i(m_i) \) whenever \( m_j = 0 \). We also impose that \( \phi_{12} \) is strictly concave in each argument, and that \( \partial^2 \phi_{12}/\partial m_i \partial m_2 \leq 0 \).\(^{18}\)

**Payoffs**

A platform’s payoff is equal to the total amount of transfers it receives (for simplicity, we assume that the marginal cost of ads is 0). An advertiser’s payoff, in case he is active on both platforms, is:

\[
u(n_1, n_2, m_1, m_2) = t_1 - t_2,
\]

where

\[
u(n_1, n_2, m_1, m_2) := \omega D_1(n_1, n_2) \phi_1(m_1) + \omega D_2(n_1, n_2) \phi_2(m_2) + \omega D_{12}(n_1, n_2) \phi_{12}(m_1, m_2)
\]

and \( t_1 \) and \( t_2 \) are the payments to platforms 1 and 2, respectively. If he only joins platform \( i \), the payoff is

\[
u(n_i, n_j, m_i, 0) - t_i = \omega \phi_i(m_i) (D_i(n_i, n_j) + D_{12}(n_i, n_j)) - t_i,
\]

since the advertiser reaches viewers only via platform \( i \). Reservation utilities are set to zero for all players.

\(^{16}\)Our results would remain unchanged if we instead assumed that if there is an overdemand for advertising slots of a platform, given the contracts offered, then actual advertising intensities are rationed proportionally for participating advertisers. We stick to the current formulation as it simplifies some of the arguments in the proofs.

\(^{17}\)A micro-foundation from previous papers works through monopolistic retail markets with identical viewers or third degree price discrimination. For example, the reservation price of all viewers equals \( \omega \), implying that each advertiser sells its product at a price of \( \omega \), appropriating all surplus from viewers. Arguably, an alternate read of this assumption is as capturing in reduced form the following idea. While getting informed about the existence of a product that matches a viewer’s taste always increases utility \textit{ex-post} (retail prices do not typically extract all surplus), the \textit{ex-ante} probability that the ‘average’ product advertised is a good ‘match’ is low enough relative to the nuisance costs. The ‘average’ advertising message is then nonetheless a disutility to viewers, that is, viewers would rather prefer ad-free content. (Hagiu and Jullien (2011) make a similar point in a different context).

\(^{18}\)A natural class of functions fulfilling these conditions is \( \phi_{12}(m_1, m_2) = \phi_{12}(m_1 + m_2) \), with \( \phi'_{12} > 0 \) and \( \phi''_{12} \leq 0 \).
Discussion of Modeling Assumptions

The functions $\phi_1$, $\phi_2$ and $\phi_{12}$ capture, in a parsimonious way, several relevant aspects of viewer behavior, platform asymmetry, and advertising technology. For example, if one platform is more effective at reaching viewers for all nonzero levels, or if viewers spend more time on one platform than on the other, this could be captured with the restriction $\phi_i(m) > \phi_j(m)$ for all $m > 0$.

Individual preferences are not necessarily independent across platforms. The model thus nests those specifications which add structure to preferences by positing a positive or negative relationship between valuations of different platforms. For example, an extreme class is the Hotelling-type spatial model with the two platforms at the opposite ends of a unit interval and viewers distributed along the interval. Thus, the Hotelling specification is captured by the above setup with the restriction $q_1 = 1 - q_2$.\footnote{Transportation costs and intercepts can be encoded in the distribution function. That is, if $k - \tau \lambda$ and $k - \tau (1 - \lambda)$ are the utility (gross of nuisance) of joining platform 1 and platform 2, respectively, with $\lambda$ uniformly distributed on $[0, 1]$, then one can compute the implied distribution on $q_1 = k - \tau \lambda$ (and similarly for $q_2$) which will depend on $\tau$.}

An important property of the demand schedules, following directly from the way we defined them, is that if $n_i$ changes but $n_j$ is unchanged, the choice of whether to join platform $j$ remains unaffected. This property contrasts either/or formulations where individuals choose one platform over the other. In our framework, if $n_i$ increases, then platform $i$ loses some single-homing and some multi-homing viewers; the single-homing viewers now become zero-homers while the multi-homers become single-homers on platform $j$. The latter implies that $\partial D_{12}/\partial n_i = -\partial D_j/\partial n_i$. We refer to this formulation as pure either/both competition.

Our assumptions on timing are meant to capture in a simple way contracting in the US and Canadian broadcast markets. On a seasonal basis, broadcasters and advertisers meet at an “upfront” event to sell commercials on the networks’ prime-time programs. At this point the networks’ supply of advertising slots is generally fixed. Due to the Nielsen rating system, which measures viewership for different programs, platforms (and advertisers) have precise viewership estimates when signing the contracts. At the upfront event, contracts that specify the number of the aired ads (so called “avails”) in exchange for a payment are then signed between broadcasters and advertisers.

The timing we assume also greatly simplifies the analysis, as the viewshipes are fixed before platforms sell their advertising slots. In Appendix 11.2 we consider a version of the model in which platforms do not announce total advertising levels, but instead offer contracts of the form $(t_i, m_i)$ to advertisers, and afterwards viewers and advertisers simultaneously decide which platform to join. We show that under some technical conditions, there exists an outcome-equivalent SPNE to that of our game. This two-stage game with posted contract offers is however much harder to analyze since a deviation by one platform leads to simultaneous changes in viewers’ and advertisers’ decisions that are influenced by each other. For this reason and due to the outcome-equivalence under certain assumptions, we stick to the easier formulation.

4 Market equilibrium

In this section we characterize the SPNE contracts offered by the platforms, and the total advertising quantities $n_1$ and $n_2$. We start by analyzing the case in which platforms compete, i.e., have different
owners. Our first observation is that after any pair of first stage announcements \((n_1, n_2)\), in any continuation equilibrium, platforms sell the same amount of advertising to all advertisers, implying \(m_i = n_i\), because the mass of advertisers equals 1. This result is a straightforward consequence of the advertising technology exhibiting diminishing returns. In turn, the price is the incremental value that advertising intensity \(n_t\) on platform \(i\) generates for an advertiser who already advertises with intensity \(n_j\) on the other platform.\(^{20}\)

**Claim 1:** In any SPNE of a game with competing platforms, given any pair of first-stage choices \((n_1, n_2)\), each platform \(i\) only offers one contract \((t_i, m_i)\). Moreover, this contract is accepted by all advertisers, and has the feature that \(m_1 = n_1, m_2 = n_2, t_1 = u(n_1, n_2) - u(0, n_2)\) and \(t_2 = u(n_1, n_2) - u(n_1, 0)\).

**Proof:** First suppose that there is a non-singleton menu of contracts \((t_i^k, m_i^k)\)\(k=1\) offered by platform \(i\) such that each of these contracts are accepted by some advertisers. Then advertisers have to be indifferent between these contracts. Let \(F(k)\) denote the cumulative density of advertisers accepting some contract \((t_i^k, m_i^k)\) for some \(k' \leq k\). Then, by strict concavity of \(\phi_i\) and \(\phi_{12}\), if platform \(i\) instead offered a single contract \((F(K)E(t_i^k), F(K)E(m_i^k))\), where the expectations are taken with respect to \(F\), each advertiser would strictly prefer to accept the contract, resulting in the same total advertising level and revenue for the platform. But then platform \(i\) could increase profits by offering a single contract \((F(K)E(t_i^k) + \varepsilon, F(K)E(m_i^k))\), for a small enough \(\varepsilon > 0\), since such a contract would still guarantee acceptance from all advertisers. The same logic can be used to establish that it cannot be in equilibrium that a single contract \((t_i, m_i)\) is offered but only a fraction of advertisers \(F(1) < 1\) accept it, since offering \((F(1) \times t_i + \varepsilon, F(1) \times m_i)\) for small enough \(\varepsilon > 0\) would guarantee acceptance by all advertisers and generate a higher revenue for platform \(i\).

The above arguments establish that the total realized advertising level on platform \(i\) is \(m_i\), the intensity specified in the single contract offered by \(i\). It cannot be that \(m_i > n_i\), since then by assumption the platform’s payoff would be negative. Moreover, since \(\phi_i\) and \(\phi_{12}\) are strictly increasing, it cannot be that \(m_i < n_i\), since then the platform could switch to offering a contract \((t_i + \varepsilon, m_i)\), which for small enough \(\varepsilon > 0\) would guarantee acceptance by all advertisers and generate a higher revenue for platform \(i\). Thus \(m_i = n_i\).

Finally, note that \(t_1 < u(n_1, n_2) - u(0, n_2)\) implies that platform 1 could charge a higher price and still guarantee the acceptance of all advertisers, while \(t_1 > u(n_1, n_2) - u(0, n_2)\) would contradict that all advertisers accept both platforms’ contracts. Hence, \(t_1 = u(n_1, n_2) - u(0, n_2)\). A symmetric argument establishes that \(t_2 = u(n_1, n_2) - u(n_1, 0)\). □

The next claim establishes a parallel result for the case in which a monopolist jointly owns both platforms. In particular, the monopolist offers a single contract, that is accepted by all advertisers.

**Claim 2:** In any SPNE of a game with jointly owned platforms, given any pair of first-stage choices \((n_1, n_2)\), the monopolist offers a single contract \((t, m_1, m_2)\). Moreover, this contract is accepted by all advertisers, and has the feature that \(m_1 = n_1, m_2 = n_2\) and \(t = u(n_1, n_2)\).

\(^{20}\)In what follows, to simplify notation we denote \(u(n_i, n_j, n_i, n_j)\) by \(u(n_i, n_j)\) and \(u(n_i, n_j, n_i, 0)\) by \(u(n_i, 0)\).
Proof: The arguments establishing that it is suboptimal for the monopolist to offer multiple different contracts, and that the contract offered by the monopolist must be accepted by all advertisers are analogous to the arguments used in the proof of Claim 1 for the competing platform case, and are hence omitted. Furthermore, profit maximization of the monopolist implies that for the single contract offered, \( m_i = n_i \) for \( i \in \{1, 2\} \).

Finally, \( t < u(n_1, n_2) \) implies that the monopolist could charge a higher price and still guarantee the acceptance of all advertisers, while \( t > u(n_1, n_2) \) would imply that no advertisers accept the contract. Hence, \( t = u(n_1, n_2) \). ■

Claims 1 and 2 imply that, since in equilibrium viewers correctly anticipate the unique continuation play following stage one, after first stage choices \((n_1, n_2)\) the resulting viewer demands are \( D_i(n_1, n_2) \), \( i = 1, 2 \), and \( D_{12}(n_1, n_2) \) in any SPNE, both in the cases of competing platforms and jointly owned platforms. As the results show, given the same pair of first stage choices, the joint revenues obtained by competing platforms in equilibrium differ from the revenue obtained by the monopolist, since in the former case platforms can only charge the incremental value of an advertiser advertising through a second channel, while the monopolist can extract the whole surplus from the advertisers. Nevertheless, the next proposition shows that the equilibrium advertising level choices are exactly the same in the two cases.

Proposition 1 (Neutrality): Equilibrium advertising levels do not depend on the competitive structure. In particular, both in the case of competing platforms and joint ownership there is a unique SPNE, and the first stage advertising level choices \((n_1, n_2)\) coincide.

Proof: Consider first the case of competing platforms. The incremental value of the contract offered by platform \( i \), which by Claim 1 is equal to platform \( i \)'s revenue, is given by the value of delivering ads to single-homing viewers (who exclusively join platform \( i \)), \( \omega \phi_i(n_i) \), plus the incremental value for the multi-homing viewers, \( \omega(\phi_{12}(n_1, n_2) - \phi_j(n_j)) \). So platform \( i \)'s profit maximization is

\[
\max_{n_i} \Pi_d^i = \omega [D_i(n_i, n_j)\phi_i(n_i) + D_{12}(n_i, n_j)(\phi_{12}(n_i, n_j) - \phi_j(n_j))] . \tag{1}
\]

The candidate equilibrium advertising levels are characterized by the following system of first-order conditions (arguments omitted for ease of exposition):

\[
\frac{\partial \Pi_d^i}{\partial n_i} = \omega \left(\frac{\partial D_i}{\partial n_i} \phi_i + D_i \frac{\partial \phi_i}{\partial n_i} + \frac{\partial D_{12}}{\partial n_i} (\phi_{12} - \phi_j) + D_{12} \frac{\partial \phi_{12}}{\partial n_i}\right) = 0, \quad i, j = 1, 2; \quad j = 3 - i. \tag{2}
\]

Our assumptions on the demand functions and on the advertising technology ensure that the second order conditions are satisfied and that there is a unique solution to the above system, and it is characterized by the above first-order conditions.

Consider now the case of joint ownership. The monopolist’s problem is

\[
\max_{n_i, n_j} \Pi^m = \omega D_1 \phi_1 + \omega D_2 \phi_2 + \omega D_{12} \phi_{12}, \quad i, j = 1, 2; \quad j = 3 - i. \tag{3}
\]

Taking the first-order condition of (3) with respect to \( n_i \) we obtain

\[
\frac{\partial D_i}{\partial n_i} \phi_i + D_i \frac{\partial \phi_i}{\partial n_i} + \frac{\partial D_{12}}{\partial n_i} (\phi_{12} - \phi_j) + D_{12} \frac{\partial \phi_{12}}{\partial n_i} = 0, \quad i, j = 1, 2; \quad j = 3 - i. \tag{4}
\]

After using \( \partial D_{12}/\partial n_i = -\partial D_j/\partial n_i \), it is evident that equations (4) and (2) are equivalent, which implies
that the solution coincides with the unique equilibrium ad levels in the competing platform case.

The following decomposition of $\Pi_i^d$, which is derived from (2) and (4), aids intuition for the neutrality result:

$$\Pi_i^d = \Pi^m - \omega \phi_j(D_j + D_{12}).$$

The above profit is reminiscent of the payoff induced by Clarke-Groves mechanisms (Clarke (1971), Groves (1973)). Each agent’s payoff is equal to the entire surplus minus a constant term—since the sum of $D_j$ and $D_{12}$ is unaffected by platform $i$’s choices in pure either or both competition—which is equal to the payoff that the other agents would get in his absence. Clarke mechanisms implement socially efficient choices, here represented by the joint monopoly solution.

An alternate way to build intuition is to inspect the first-order conditions for an optimum. When marginally increasing $n_i$, a monopolistic platform loses some multi-homing viewers, who become single-home viewers on platform $j$. With the first kind of viewers the monopolist loses $\omega \phi_{12}$, while with second he gains $\omega \phi_j$. In duopoly, when a platform increases $n_i$, it loses some multi-homing viewers whose values are $\omega(\phi_{12} - \phi_j)$. But this implies that the trade-offs in both market structures are the same.21

The neutrality result is also reminiscent of common agency models (e.g., Bernheim and Whinston, 1985 and 1986), that predict the same outcomes in monopoly and in duopoly. However, common agency models feature a single agent who contracts with multiple principals instead of a continuum of agents, as in our framework. In particular, if there is only a single advertiser—or, equivalently, if all advertisers can coordinate their choices22—even in models featuring either/or competition, the equilibrium advertising level is the same with duopoly and with joint monopoly. To see this consider the case in which viewers join either platform $i$ or $j$, implying that $D_{12} = 0$. If there is only a single advertiser, the transfer that platform $i$ can charge to make the advertiser accept is the incremental value of the platform, i.e., $u(n_{d1}^i, n_{d2}^j) - u(0, n_{dj}^j)$. In the either/or framework, $u(n_{d1}^i, n_{d2}^j) = \omega D_1(n_{d1}^i, n_{d1}^j) \phi_1(n_1) + \omega D_2(n_{d1}^i, n_{d2}^j) \phi_2(n_2)$ while $u(0, n_{dj}^j) = \omega D_j(0, n_{dj}^j) \phi_j(n_j)$. Hence,

$$\Pi_i^d = \omega D_1(n_{d1}^i, n_{d1}^j) \phi_1(n_1) + \omega D_2(n_{d1}^i, n_{d2}^j) \phi_2(n_2) - \omega D_j(0, n_{dj}^j) \phi_j(n_j).$$

The first two terms are equivalent to a monopolist’s profit, while the last term is independent of $n_{d1}^i$. Therefore, the first-order conditions for monopoly and duopoly coincide and neutrality obtains. It follows that competition is disabled in common agency models independent of the agent being able to single-or to multi-home. In contrast, we show that with a continuum of advertisers neutrality obtains in the either/both framework (but not in the either/or framework, like in Anderson and Coate (2005)).

**Extensions**

I. Simpler contract space. The neutrality result remains intact if platforms cannot offer general contracts, but can only charge a per-unit price plus an entrance fee. In this case, the unique equilibrium advertising levels are still characterized by (2) in monopoly and in duopoly. To see this, first consider

---

21Note that in both cases increasing $n_i$ also implies losing some single-homing viewers on platform $i$. But the loss from this is exactly the same for the monopolist and the duopolist.

22For an analysis of consumer coordination in platform competition in a setting with positive network externalities, see Ambrus and Argenziano (2009).
the monopoly case. For the monopolist the optimal entrance fee will be \( F_{12} = \omega (D_1 \phi_1 + D_2 \phi_2 + D_{12} \phi_{12}) - p_1 n_1 - p_2 n_2 \) to ensure participation of all advertisers. Hence the monopolist’s profit is \( \Pi_{12} = F_{12} + p_1 n_1 + p_2 n_2 = \omega (D_1 \phi_1 + D_2 \phi_2 + D_{12} \phi_{12}) \). Therefore, the optimal \((n_1, n_2)\) pair is the same as in our original model. For a duopolist a similar argument applies. The optimal fee for platform \( i \) is \( F_i = \omega (D_i \phi_i + D_{12} (\phi_{12} - \phi_j)) - p_i n_i \), implying that platform \( i \)'s profit is \( \Pi_i = F_i + p_i n_i = \omega (D_i \phi_i + D_{12} (\phi_{12} - \phi_j)) \). This is the same maximization problem as in our original setup. This modified pricing structure strips the monopolist from the ability to bundle, which is a realistic assumption in some settings. However, the monopolist can extract all surplus via the participation fee, and hence bundling is not necessary for full surplus extraction.

II. More than two platforms. In Appendix 11.2 we show that the neutrality result extends to any finite number of platforms, and to any platform ownership structure. We do not feature this generalization in the main text because the arguments required are conceptually the same as those used in the proof of Proposition 1, but the notation is more cumbersome. This generalization allows for a reinterpretation of the model in terms of programs, broadcasted by different distributors/channels, as in the case of digital TV with downloadable content. A viewer who watches several programs (at different times) can thereby connect several times to the same channel. Moreover, this reinterpretation of the model facilitates explicitly modeling the time a viewer spends watching a given channel since it is implied by the programs that the viewer watches on this channel.

III. Heterogeneous advertisers. This extension requires a completely different methodology, as platforms want to offer a menu of contracts, catering different types of advertisers. We consider this extension formally in Section 9, and show that with some qualifications the neutrality result carries over.

5 Welfare analysis

Industry observers often claim that the market provides an inefficiently high quantity of advertising. To address this concern we proceed by characterizing the socially optimal allocation. As mentioned, \( q_i - \gamma n_i \) is the utility of a single-homing viewer of platform \( i \) and \( q_1 - \gamma n_1 + q_2 - \gamma n_2 \) is the utility of a multi-homing viewer. Social welfare is given by

\[
W = \int_{\gamma n_1}^{\infty} \int_{0}^{\gamma n_2} (q_1 - \gamma n_1) h(q_1, q_2) dq_2 dq_1 + \int_{\gamma n_1}^{\infty} \int_{\gamma n_2}^{\infty} (q_2 - \gamma n_2) h(q_1, q_2) dq_2 dq_1 + \int_{\gamma n_1}^{\infty} \int_{\gamma n_2}^{\infty} (q_1 - \gamma n_1 + q_2 - \gamma n_2) h(q_1, q_2) dq_2 dq_1 + \omega D_1 \phi_1 + \omega D_2 \phi_2 + \omega D_{12} \phi_{12}.
\]

Comparing the equilibrium advertising level with the socially efficient advertising level we obtain the following:

**Proposition 2:** Equilibrium advertising levels are inefficiently high.

**Proof:** See Appendix 11.1.

To grasp this result, it is useful to consider the incentives of the joint monopoly platform. Note that under our assumptions the monopolist fully internalizes advertisers’ welfare. On the contrary, it does not internalize viewers’ welfare. More precisely, it only cares about viewers’ utilities inasmuch as they contribute to advertising revenue, while the nuisance costs from advertising are not taken into account.
This leads to inefficiently high advertising levels. By Proposition 1 competing platforms implement the same allocation.

Proposition 2 should be interpreted with caution. The overprovision result hinges on the assumption that advertisers are homogeneous. If advertisers are heterogeneous with respect to their product qualities, an extensive margin comes into play in addition to the intensive margin considered so far. This extensive margin arises because, as in previous literature, a platform owner trades off the marginal revenue from an additional advertiser with the revenues from inframarginal advertisers. This effect coupled with our result can either lead to socially excessive or socially insufficient advertising levels.

Propositions 1 and 2 together imply that with either/both type competition, advertising levels tend to be inefficiently high as competition does not contribute to alleviate this distortion. This is consistent with the existence of regulatory “caps” or ceilings on the number of commercials per hour in many countries, suggesting concerns of overprovision of advertising. We note that our conclusions differ from those obtained in models with either/or competition. For instance, in Anderson and Coate (2005) competition for viewers always reduces advertising levels relative to monopoly, which can lead to inefficiently low advertising levels, even with homogeneous advertisers.

6 Entry

In this section we consider the effect that entry of an additional platform has on the advertising level of an incumbent platform. Intuitively, an unchallenged monopoly incumbent is able to fully extract the advertisers’ surplus through the transfer, while entry will limit its payoff to the incremental value, as discussed. Formally we need to compare the solution of

\[ n_i^{s.m.} := \arg \max_{n_i} d_i(n_i) \phi_i(n_i), \]  

where \( d_i(n_i) \equiv \text{Prob}\{q_i - \gamma n_i \geq 0\} \), to the duopoly solution characterized in Section 4.\(^{23}\) From the incumbent’s perspective, entry implies sharing some of its previously exclusive business.\(^{24}\) The question is how entry affects the basic trade-off faced by the incumbent underlying the problem in (5). For this purpose it is useful to rewrite the duopolist’s payoff as if all viewers were exclusive plus a (negative) correction term that accounts for the fact that \( i \) can only extract the incremental value from those viewers who are shared after entry:

\[ n_i^d := \arg \max_{n_i} d_i(n_i) \phi_i(n_i) + D_{12}(n_1, n_2) (\phi_{12}(n_1, n_2) - \phi_j(n_j) - \phi_i(n_i)). \]  

First consider problem (5). Its solution is characterized by the first-order condition

\[ \frac{\partial \phi_i}{\partial n_i} d_i + \frac{\partial d_i}{\partial n_i} \phi_i = 0. \]  

\(^{23}\)Here we adopt the convention that \( i \) denotes the incumbent platform. The superscript \( s.m. \) reads ‘single platform monopolist,’ and is meant to avoid confusion with the joint multi-platform monopolist solution considered so far.

\(^{24}\)This is because when an additional platform enters the market, then the total demand faced by platform \( i \) does not change, given \( n_i \) stays constant. However, some of its previously exclusive viewers will now be served by both platforms, that is \( d_i(n_i) = D_i(n_i, n_j) + D_{12}(n_1, n_2) \).
When increasing its quantity \( n_i \), platform \( i \) trades off gains on inframarginal viewers due to increased reach with revenues from marginal viewers who switch off due to the increased advertising. If we introduce the elasticities of the total demand \( d_i \) and of the function \( \phi_i \) with respect to \( n_i \),

\[
\eta_{d_i} := -\frac{\partial d_i}{\partial n_i} n_i \quad \text{and} \quad \eta_{\phi_i} := \frac{\partial \phi_i}{\partial n_i} n_i,
\]

then the incumbent’s pre-entry optimal quantity can be characterized by the following simple and intuitive condition:

\[
\eta_{\phi_i} = \eta_{d_i}.
\]

Consider now problem (6). Once entry occurred, condition (7) should be augmented to account for the fact that some of the previously exclusive business is shared:

\[
\frac{\partial \phi_i}{\partial n_i} d_i + \frac{\partial d_i}{\partial n_i} \phi_i + D\frac{\partial(\phi_{12} - \phi_i - \phi_j)}{\partial n_i} + \frac{\partial D_{12}}{\partial n_i}(\phi_{12} - \phi_i - \phi_j) = 0. \tag{8}
\]

Since the advertising technology is concave, i.e., \( \frac{\partial \phi_i}{\partial n_i} > \frac{\partial \phi_{12}}{\partial n_i} \), the third term of (8) is negative. It captures the fact that the incremental reach due to an additional ad on shared viewers is also small relative to that of exclusive viewers. Ceteris paribus, this gives the platform an incentive to decrease \( n_i \). By contrast, the third term of (8) is positive because \( \frac{\partial D_{12}}{\partial n_i} < 0 \) and \( \phi_i - \phi_j > \phi_{12} \). The term captures the fact that due to incremental pricing, shared viewers are less valuable. The foregone benefit from losing such viewers is small relative to that of exclusive viewers. Ceteris paribus, this gives the platform an incentive to increase \( n_i \). The overall effect is ambiguous and, as the next proposition shows, the effect that dominates depends on the relative elasticity of multi-homing versus single-homing viewers, normalized by the elasticity of the relative reach. Defining

\[
\eta_{D_{12}} := -\frac{\partial D_{12}}{\partial n_i} \frac{n_i}{D_{12}} \quad \text{and} \quad \eta_{\phi_i + \phi_j - \phi_{12}} := \frac{\partial (\phi_i + \phi_j - \phi_{12})}{\partial n_i} \frac{n_i}{\phi_i + \phi_j - \phi_{12}},
\]

respectively as the elasticity of overlapping viewers with respect to \( n_i \) and the elasticity of \( \phi_i + \phi_j - \phi_{12} \) with respect to \( n_i \), we get the following characterization.

**Proposition 3:** Platform entry increases (decreases) the incumbent platform’s advertising level if and only if

\[
\frac{\eta_{D_{12}}}{\eta_{d_i}} > (<) \frac{\eta_{\phi_i + \phi_j - \phi_{12}}}{\eta_{\phi_i}}, \tag{9}
\]

where all functions are evaluated at \( n_i = n_i^{s.m.} \) and \( n_j = n_j^d \).

**Proof:** See Appendix 11.1.

To build intuition, consider the simplest case in which the two platforms are symmetric, i.e., \( d_i(n) = d_j(n) \), \( \phi_i(n) = \phi_j(n) \), and \( \phi_{12}(n_1, n_2) = \phi_{12}(n_2, n_1) \) for all \( n, n_1, \) and \( n_2 \), and suppose that post-entry both levels are set at the optimal pre-entry levels: \( n_j = n_i = n_i^{s.m.} \). Can these quantities constitute an equilibrium?

First, recall that overlapping viewers receive advertising messages from both platforms. So at these levels the amount of advertising they receive is duplicated. Ceteris paribus, decreasing marginal returns give an incentive to scale back advertising on platform \( i \), whose marginal contribution to the advertisers’
retail profits drops as a result of such duplication. Note that this *duplication effect* is a straightforward consequence of the fact that viewers multi-home.

The second and arguably more subtle effect that we distinguish is what we label as *business-sharing effect*. Recall that \( d_i(n_i) = D_i(n_i) + D_{12}(n_1, n_2) \), so that it is possible to decompose the total variation in demand due to a small increase in \( n_i \) as \( \frac{\partial D_i}{\partial n_i} + \frac{\partial D_{12}}{\partial n_i} \). When the monopolist is unchallenged, it is wary of the total variation of \( d_i \) regardless of how this variation is spelled out. However, when an additional platform is present, the (former) monopolist distinguishes between the two sources of variation, i.e., \( \frac{\partial D_i}{\partial n_i} \) and \( \frac{\partial D_{12}}{\partial n_i} \). The important insight is that the opportunity cost of losing shared business is lower than the cost of losing exclusive viewers, as the former viewers will be informed with positive probability on platform \( j \). Therefore, the cost from lost demand is relatively small. Ceteris paribus, this leads to a higher advertising level in equilibrium. In particular, as can be seen from (9), if \( |\frac{\partial D_{12}}{\partial n_i}| \) is large compared to \( |\frac{\partial D_i}{\partial n_i}| \), implying that \( \eta D_{12} \) is large compared to \( \eta d_i \), advertising levels in duopoly increase relative to monopoly.

We conclude this section by investigating the condition in Proposition 3 for specific advertising technologies. In particular, suppose that the functional form is either a power function, or a negative exponential function (also known in the literature as the Butter’s (1977) technology):

\[
\phi_i(n_i) = n_i^{1/a} \quad \text{and} \quad \phi_{12}(n_1, n_2) = (n_1 + n_2)^{1/a},
\]

\[
\phi_i(n_i) = 1 - e^{-bn_i} \quad \text{and} \quad \phi_{12}(n_1, n_2) = 1 - e^{-b(n_1 + n_2)}.
\]

Since \( \phi \) is increasing and concave, the parametric restrictions are \( a \in (1, \infty) \) and \( b \in (0, \infty) \). For \( a, b \to \infty \), the advertising technology resembles the case in which overlapping viewers are of zero value, because then \( \phi_i(n_i) = 1 \) and \( \phi_{12}(n_1, n_2) = 1 \).

Suppose that platforms are symmetric. It is then easy to check that for the advertising technology given by (i), we obtain \( \eta_{\phi_i + \phi_j - \phi_{12}}/\eta_{\phi_i} = 1/2 \), while for the exponential advertising technology form (ii), we obtain \( \eta_{\phi_i + \phi_j - \phi_{12}}/\eta_{\phi_i} = 1 \) leading to the following result.

**Claim 3:** Suppose platforms are symmetric and that the advertising technology is given by either (i) or (ii). Then the advertising level increases with entry if and only if \( \eta_{D_{12}}/\eta_{d_i} > 1/2 \) or \( \eta_{D_{12}}/\eta_{d_i} > 1 \), respectively.

### 7 Viewer Preference Correlation

The generality of our framework allows us to ask various questions regarding the effects of correlation between viewers’ preferences. For instance, what is the effect in equilibrium if an entrant platform mostly appeals to the same viewers as the incumbent? Would this entrant be more or less profitable than a platform that caters to those who find the incumbent unappealing? We note that comparable results cannot be obtained in existing models of either/or competition, as these models draw either on Hotelling spatial models or assume a representative viewer. In the first case the correlation between viewers’ preferences is assumed to be perfectly negative, i.e., the viewer who likes platform \( i \) most likes platform \( j \) least, while in the second case viewers are the same by assumption.

A key observation is that a higher correlation coefficient, ceteris paribus, increases the overlap \( D_{12} \),
while the total demand for each platform $d_i$ is unchanged.\textsuperscript{25} So, for example, if $q_1 - \gamma n_1 > 0$ then an increase in the correlation between $q_1$ and $q_2$ makes it more likely that $q_2 - \gamma n_2 > 0$ as well. Therefore, when stating that viewer preferences are more or less correlated, we mean a higher or lower $D_{12}$ for a given $d_i$ and $d_j$.

We simplify the analysis by focusing on the case of linear demand, that is, we assume $\partial^2 D_i/\partial n_i^2 = \partial^2 D_{12}/\partial n_i^2 = 0$. We maintain the level of generality on the supply side, allowing for a general advertising technology and asymmetric platforms. The next result states that the effect of an increase in the correlation in viewer’s preferences on equilibrium advertising levels is unambiguously negative for the case of linear demand.

**Proposition 4:** Suppose that $\partial^2 D_i/\partial n_i^2 = \partial^2 D_{12}/\partial n_i^2 = 0$. Then the equilibrium advertising levels are decreasing in the correlation in viewers’ preferences.

**Proof:** See Appendix 11.1.

For an intuition behind the result, note that an increase in correlation is equivalent to a change in the demand composition, whereby a higher fraction of platform $i$’s total demand is comprised of overlapping viewers. As a consequence, the additional value of an ad is relatively small because a large mass of viewers is already exposed to this ad on the other platform - this is the duplication effect. In addition, by lowering its advertising level, a platform changes its viewer composition such that most of its new viewers are single-homers. That is, most of the total demand variation comes from viewers who do not join platform $j$. As pointed out before, these exclusive viewers are relatively valuable. These two effects in combination give platforms an incentive to set a low advertising level.

We can now analyze how the viewer preference correlation affects the entry effects. We know that for given advertising levels, an increase in the viewer preference correlation leads to an increase in $D_{12}$ and a fall in $D_i$ by the same value. Also, an increase in $D_{12}$ leads to a fall in $\eta_{D_{12}}$ in the case of linear demand while $\eta_{d_i}$ stays unchanged. This implies that the left-hand side of (9) falls, while the right-hand side of (9) does not change, implying that $n_i^d < n_i^{e,m}$ becomes ‘more likely.’ We can therefore state the following result:

**Proposition 5:** Suppose that $\partial^2 D_i/\partial n_i^2 = \partial^2 D_{12}/\partial n_i^2 = 0$. Then, platform entry is more likely to lead to an increase in advertising levels if the content of the entering platform is less correlated with the incumbent’s content.

To develop intuition we further restrict attention to the following class of specifications. Suppose viewer types are distributed on the unit square. A fraction $1 - \lambda$ of viewers is uniformly distributed on this square. The remaining fraction $\lambda$ is uniformly located on the 45-degree line. This is illustrated in the left-hand side of Figure 1. So varying $\lambda$ allows us to express different degrees of correlation ranging from independent preferences if $\lambda = 0$ to perfect positive correlation if $\lambda = 1$.\textsuperscript{26} For simplicity, assume $\gamma = 1$, implying that a viewer joins platform $i$ if $q_i - n_i \geq 0$. Likewise, we can express negative correlation by

\textsuperscript{25}Recall that the total demand of platform $i$ depends only on the marginal distribution.

\textsuperscript{26}The example does not satisfy the smoothness requirement spelled out in Section 3. However, it can be arbitrarily approximated by smooth distributions, and the qualitative conclusions would be the same for those nearby smooth distributions.
distributing a mass $\lambda$ on the line from $(0,1)$ to $(1,0)$ (rather than on the line from $(0,0)$ to $(1,1)$). The larger $\lambda$ is, the more negative is the preference correlation. This scenario is displayed by the right-hand side in Figure 1. In fact, there is a one-to-one mapping between $\lambda$ and the correlation coefficient $\rho$ of the probability distribution for each diagram, that is, $\lambda$ spans all values of $\rho$ in $[0,1]$ in the left diagram and all values of $\rho$ in $[-1,0]$ in the right diagram.

![Figure 1: Positive (left) and negative (right) correlation](image-url)

Finally, assume $\phi_i(n_i) = 1 - e^{-n_i}$ and $\phi_{12}(n_1, n_2) = 1 - e^{-(n_1+n_2)}$, which implies that $\phi(\cdot)$ is strictly concave. Proposition 3 states that under this assumption the business sharing effect dominates if and only if most of the variation comes from overlapping viewers: $\eta_{D_{12}} > \eta_{d_i}$. Now consider an increase in correlation. This increase moves the mass towards symmetric valuations, implying that there are more overlapping viewers. This implies that the denominator of $\eta_{D_{12}}$ increases, i.e., the elasticity of these viewers falls. Therefore, the business sharing effect is ‘less likely’ to dominate and advertising levels are more likely to fall after entry. This translates to the following result:

**Proposition 6:** Platform entry results in a change in the quantity of advertising supplied in equilibrium by the incumbent whose sign is the opposite sign of the correlation coefficient $\rho$:

$$\text{sign}(n^d_i - n^{s,m}_i) = \text{sign}(\eta_{D_{12}} - \eta_{d_i}) = -\text{sign}(\rho).$$

where all functions are evaluated at $n_i = n^{s,m}_i$ and $n_j = n^d_j$.

**Proof:** See Appendix 11.1.

Our analysis is therefore able to explain why the entry of FOX News has led to an increase in the advertising level of other stations like e.g., CNN, for which it is likely that viewer preferences are negatively correlated. However, for platforms with positive correlation, our model predicts the opposite. In Appendix 11.3 we provide a simple empirical analysis based on data of the U.S. cable TV industry, which is consistent with the prediction of our model.

Finally, we can consider the content choice of the platforms. It is evident from the profit function,

---

27Although extending the above result to more flexible demand systems is beyond the scope of this paper, we numerically simulated the demand from a jointly bivariate normal distribution of $(q_1,q_2)$ and obtained the same result. That is, the sign of the incumbent’s change in advertising levels is fully characterized by the sign of the correlation coefficient that parametrizes the joint bi-variate distribution.
\[ \Pi'_i = \omega [D_i \phi_i + D_{12} (\phi_{12} - \phi_j)] \], that for given advertising levels, the platform prefers exclusive to overlapping viewers. Since we know that increasing the viewer preference correlation shifts viewers from \( D_i \) to \( D_{12} \), this increase is likely to be detrimental for platform’s profit. Hence, platforms prefer viewers preferences to be negatively correlated. Although content is exogenous in the model, a natural prediction based on our analysis is that if platforms have some freedom to choose their content, they will differentiate their content from that of their rivals. Although this conclusion is similar to standard models of differentiation, the intuition is different. In the standard model, firms differentiate their content to reduce competition. In our case, there is no competition for viewers but platforms choose negative correlation to reduce overlapping viewships.

8 Viewer Pricing

In this section we allow platforms to charge viewers. In particular, we are interested if neutrality carries over the the case of viewer pricing and how viewer pricing affects the results on correlation and entry.

Let \( p_i \) denote the viewer price at platform \( i \). Platforms set the prices in the first stage, i.e., at the same time that they announce total advertising levels and before viewers decide which platform(s) to join. Otherwise, the model is the same as described in Section 3. In line with the literature, we restrict viewer prices to be non-negative, since implementing subsidies is problematic in practice.\(^{28}\) The utility of a viewer of type \( q_i \) from joining platform \( i \) is then given by

\[ q_i - \gamma n_i - p_i. \]

The demand schedules are now given by:

- Multi-homers: \( D_{12} := \text{Prob}\{q_1 - \gamma n_1 - p_1 \geq 0 ; q_2 - \gamma n_2 - p_2 \geq 0\} \),
- Single-homers: \( D_1 := \text{Prob}\{q_1 - \gamma n_1 - p_1 \geq 0 ; q_2 - \gamma n_2 - p_2 < 0\} \),
- Single-homers: \( D_2 := \text{Prob}\{q_1 - \gamma n_1 - p_1 < 0 ; q_2 - \gamma n_2 - p_2 \geq 0\} \),
- Zero-Homers: \( D_0 := 1 - D_1 - D_2 - D_{12} \).

We first compare the advertising levels in duopoly and in joint monopoly, and verify that the neutrality result carries over to this setting.

**Proposition 7:** The neutrality result, that equilibrium advertising levels are the same in duopoly and in joint monopoly, carries over to the specification that allows for viewer pricing.

**Proof:** The profit function of platform \( i \) in duopoly is

\[ \Pi'_i = \omega (D_i \phi_i + D_{12} (\phi_{12} - \phi_j)) + p_i (D_i + D_{12}). \]

Differentiating with respect to \( n_i \) and \( p_i \), we obtain the first-order conditions:

\[ \frac{\partial \Pi'_i}{\partial n_i} = \omega \left[ \frac{\partial D_i}{\partial n_i} \phi_i + D_i \frac{\partial \phi_i}{\partial n_i} + \frac{\partial D_{12}}{\partial n_i} (\phi_{12} - \phi_j) + D_{12} \frac{\partial \phi_{12}}{\partial n_i} \right] + p_i \left( \frac{\partial D_i}{\partial n_i} + \frac{\partial D_{12}}{\partial n_i} \right) = 0, \] \hspace{1cm} (10)

\(^{28}\)For example, as Anderson and Coate (2005) point out, it is impossible to know whether the viewer is paying attention, even if monitoring viewer behavior is possible.
and
\[ \frac{\partial \Pi^d}{\partial p_i} = \omega \left[ \frac{\partial D_i}{\partial p_i} \phi_i + \frac{\partial D_{12}}{\partial p_i} (\phi_{12} - \phi_j) \right] + D_i + D_{12} + p_i \left( \frac{\partial D_i}{\partial p_i} + \frac{\partial D_{12}}{\partial p_i} \right) = 0. \] (11)

By our assumptions on the viewer demand functions, and on the advertising technology, the second-order conditions are satisfied and the resulting equilibrium is unique. Therefore, equations (10) and (11) determine the equilibrium advertising levels and viewer charges in duopoly.

The profit function of a joint monopolist is
\[ \Pi^m = \omega (D_1 \phi_1 + D_2 \phi_2 + D_{12} \phi_{12}) + p_1 D_1 + p_2 D_2 + (p_1 + p_2) D_{12}. \]

Differentiating this function with respect to \( n_i \) and \( p_i \), and using that \( \partial D_j / \partial n_i = -\partial D_{12} / \partial n_i \) and \( \partial D_j / \partial p_i = -\partial D_{12} / \partial p_i \), it is easy to check that we obtain the same first-order conditions as in (10) and (11), implying that advertising levels in joint monopoly and duopoly are again the same. ■

The result shows that viewer pricing does not change the similarity in the trade-off for a joint monopolist and for a duopolist. So, neutrality does not depend on the number of pricing instruments but is inherent in either/both competition.

Although neutrality carries over, the advertising level changes when viewer pricing is allowed. Since viewer pricing provides platforms with an additional revenue source, platforms substitute some advertising revenues for viewer revenues, thereby reducing the advertising level. Turning to the effects of viewer preference correlation, it is of interest to explore if viewer pricing affects our result of Proposition 4. The next proposition shows that this is not the case:

**Proposition 8:** The equilibrium advertising levels are decreasing in the viewer preference correlation. In addition, if \( \partial^2 D_i / \partial n_i^2 = \partial^2 D_{12} / \partial n_i^2 = 0 \), then the equilibrium viewer price is increasing in the viewer preference correlation.

**Proof:** See Appendix 11.1.

The first statement establishes that our prior result that advertising levels fall with correlation carries over to the situation with viewer pricing. The result is even stronger in this case because it holds independently of the shape of the viewer demand function. The second statement shows that viewer prices react in the opposite way compared to advertising levels (for linear demand specifications). As explained before, when the viewer preference correlation increases, advertising on a platform becomes less valuable since more viewers were already exposed to advertising on the other platform. This effect is present with or without viewer pricing, implying that advertising levels fall in equilibrium. However, platforms can now compensate for this reduction of advertising revenue by increasing viewer prices. Since the value to a platform in our pure either/both model is the same for single-homing and multi-homing viewers, the optimal pricing decision is not affected by the viewer preference correlation, as long as advertising levels are constant. However, since advertising levels fall, viewers value platforms more, implying that platforms charge higher prices. Hence, we predict that platforms with higher correlation broadcast less advertising but charge higher viewer fees.

We now turn to the effects of entry. Here we obtain the following result:

**Proposition 9:** With viewer pricing, entry weakly decreases the incumbent platform’s advertising levels. Moreover, if \( \partial^2 D_i / \partial n_i^2 = \partial^2 D_{12} / \partial n_i^2 = 0 \), entry weakly increases the viewer price charged by the incumbent.
Proof: See Appendix 11.1.

The result differs from the one of Proposition 3, which states that without viewer pricing advertising levels can rise or fall with entry. With viewer pricing, there is a duplication effect and a countering business-sharing effect as well. However, the duplication effect dominates. To see this compare the optimal solution of a a single-platform monopolist to the one of a monopolist controlling both platforms. The duplication effect only affects the latter. Therefore, to keep viewer demand constant, the two-platform monopolist reduces advertising levels and increases viewer prices. The business-sharing effect is then of second-order because it only arises when total viewer demand varies. Therefore, the duplication effect dominates, leading to lower advertising levels and higher viewer prices after entry.

Lastly, we can compare social welfare with viewer pricing to the case without viewer pricing. As Anderson and Coate (2005), we find that viewer pricing can lead to a rise or a fall in welfare. Welfare can rise if viewers strongly dislike advertising, because advertising levels fall. However, the full cost to viewers is larger with viewer pricing, implying that more viewers switch off, which can lead to a reduction in welfare. The distributional consequences of viewer prices are that viewer utility falls and also advertising revenues fall. Hence, allowing for viewer pricing redistributes revenue from viewers and advertisers to platforms. We formally establish these results in Appendix 11.2.

9 Heterogeneous Advertisers

Here we discuss how the neutrality result extends to advertisers with heterogeneous product values, as in Anderson and Coate (2005). First, it is evident that neutrality also holds if platforms can offer a menu of advertising intensities and payments and can perfectly discriminate between advertisers. In that case, the result is similar to the one for the case of homogeneous advertisers. The analysis is less straightforward if platforms cannot perfectly discriminate, i.e., when $\omega$ is private information to each advertiser, because we need to determine the optimal screening contracts offered by platforms, instead of only a single contract.

Below we show that neutrality still holds if we restrict attention to the class of contracts that only allows the monopolist to bundle advertising on different platforms through a joint entrance fee (and aside from the entrance fee, the payment from an advertiser for advertising on platform $i$ is only contingent on its advertising intensity on platform $i$, not on platform $j$). We note that neutrality does not necessarily hold when a joint monopolist can offer bundled contracts that specify a single transfer in exchange for an advertising intensity on each platform, because in this case an advertiser cannot report different types to the two platforms. This may change the advertiser’s outside option and affect the optimal allocation induced by the monopolist.

The main takeaway from this extension is that even when there is a lot of heterogeneity among advertisers, the neutrality result remains valid as long as a monopolist owning both platforms is constrained in the extent of bundling of advertising intensities at different platforms. However, when there is both heterogeneity among advertisers and no constraints on bundling, the neutrality result does not necessarily hold. The analysis reveals that in this case it is not competition per se that changes advertising levels. Rather, the advertisers’ limited possibility to report their types is responsible for the potential of different outcomes in monopoly and duopoly. If this capacity is the same in monopoly and duopoly—as is the case when each platform owner can offer a contract depending only on this platform’s advertising intensity - neutrality is restored, i.e., competition has no bite in reducing advertising levels.
To analyze advertiser heterogeneity, consider the following modification of our benchmark model: The value of informing a viewer, $\omega$, is distributed according to a smooth c.d.f. $F$ with support $[\underline{\omega}, \overline{\omega}]$, $0 < \underline{\omega} \leq \overline{\omega}$, that satisfies the monotone hazard rate property. The value $\omega$ is private information to each advertiser. The timing of the game is the same as before. In the first stage, each platform $i$ announces its total advertising level $n_i$. Afterwards, viewers decide which platform to join. Given these decisions, each platform owner offers a menu of contracts consisting of an entrance fee and a price schedule $t_i : m_i \rightarrow \mathbb{R}$ defined over a compact set of advertising levels $m_i \equiv [0, \overline{m}]$, for each platform $i$ owned by him. In particular, a monopolist platform owner can set a joint entrance fee $t_0$, paying which is a pre-requisite for an advertiser to advertise on any of the platforms. In the case of duopoly, without loss of generality we can assume that the entrance fees set are 0, since a positive entrance fee can be added as a constant to the starting fees.

We now characterize platform $i$’s best reply in duopoly competition, that is, the price schedule $t_i$ that maximizes its payoff given $t_j$. With an abuse of notation we still use $\omega u(m_1, m_2, n)$ to denote the surplus of advertiser $\omega$ from advertising intensities $(m_1, m_2)$. The overall utility of an advertiser depends on the price schedules in addition to the surplus. If $m_i(\omega)$ denotes the optimal quantity chosen by type $\omega$, then platforms $i$’s problem, given its rival’s price schedule $t_j$, is:

$$\Pi = \max_{t_i(\cdot)} \int_{\underline{\omega}}^{\overline{\omega}} t_i(m_i(\omega))dF(\omega).$$  \hspace{1cm} (12)

The above can be expressed as a standard screening problem:

$$\Pi = \max_{\omega_0, m_i(\omega)} \int_{\omega_0}^{\overline{\omega}} t_i(m_i(\omega))dF(\omega)$$

subject to $m_i(\omega) = \arg\max_{m_i} v^d_i(m_i, \omega, n) - t_i(m_i),$

$v^d_i(m_i(\omega), \omega, n) - t_i(m_i(\omega)) \geq 0$ for all $\omega \geq \omega_0,$

$\int_{\underline{\omega}}^{\overline{\omega}} m_i(\omega')dF(\omega') \leq n_i,$

where $v^d_i(m_i, \omega, n) := \max_y \omega u(m_i, y, n) - t_j(y) - (\max_y \omega u(0, y', n) - t_j(y'))$, with $u(m_i, y, n) := D_i(n_1, n_2) \phi_i(m_i) + D_j(n_1, n_2) \phi_j(y) + D_{12}(n_1, n_2) \phi_{12}(m_i, y)$, denotes the net value of advertising intensity $m_i$ on platform $i$ to type $\omega$. This is the value of contracting with $i$ given the price schedule of the rival. It is the maximum value minus the outside option of dealing with $j$ exclusively. Note that in any pure strategy equilibrium platform $i$ behaves as a monopolist facing a mass one of advertisers with indirect utility functions $v^d_i$. Provided that this function satisfies standard regularity conditions in the screening literature, we can apply the canonical methodology developed by Mussa and Rosen (1978) or Maskin and Riley (1984) to characterize $i$’s best reply. As in Martimort and Stole (2009), $v^d_i$ is labeled regular if it is continuous, monotone in $\omega$, and satisfies strict increasing differences in $(m, \omega)$. Our assumptions on the viewer demands $D_i(n_1, n_2)$ and $D_{12}(n_1, n_2)$ and on the advertising technology $\phi_i(m_i)$ and $\phi_{12}(m_1, m_2)$ ensure that $v^d_i$ is continuous and increasing in $\omega$. It also has strict increasing differences in $(m, \omega)$. An equilibrium $(t^d_1, t^d_2)$ is said to be regular if the induced indirect utility functions are regular.29

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29 In the proof of Proposition 10, the corresponding virtual surplus is given by $v^d_i(m_i, \omega, n) -$
We contrast platform \(i\)'s best reply with the optimal price schedule that a multi-platform monopolist would choose for platform \(i\) given an arbitrary price schedule \(t_j\). The monopolist’s profits are:

\[
\Pi = \max_{t_i(\cdot), t_j(\cdot), t_0} \int_{\omega} t(m_1(\omega), m_2(\omega)) dF(\omega),
\]

with

\[
t(m_1(\omega), m_2(\omega)) = \begin{cases} 
  t_0 + t_1(m_1(\omega)) + t_2(m_2(\omega)) & \text{if } (m_1(\omega), m_2(\omega)) \neq (0, 0) \\
  0 & \text{otherwise}.
\end{cases}
\]

Once more, it is possible to derive the induced indirect utility function \(v^m_i(m_i, \omega, n) = \max_y \omega u(m_i, y, n) - t_y(y) - t_0 - \sup \{\max_{y'} \omega u(0, y', n) - t_y(y') - t_0, 0\}\) and express the above as a standard screening problem as follows:

\[
\Pi = \max_{\{t_i(\cdot)\}_{i=1}^2, \{m_i(\cdot)\}_{i=1}^2, t_0, \omega_0} \int_{\omega_0} t(m_1(\omega), n_2(\omega)) dF(\omega)
\]

subject to

\[
\begin{align*}
  m_i(\omega) &= \arg \max_{n_i} v^m_i(m_i, \omega, n) - t_i(m_i) \\
v^m_i(m_i(\omega), \omega, n) - t_i(m_i(\omega)) &\geq 0 \text{ for all } \omega \geq \omega_0 \\
\int_{\omega_0} m_i(\omega') dF(\omega') &\leq n_i.
\end{align*}
\]

A solution to the monopoly problem \((t_1^m, t_2^m)\) is said to be regular if the induced indirect utility functions are regular. Finally, we assume that the profit function \(\int_{\omega_0} \Lambda^m(m_i(\omega), \omega, n) dF(\omega)\) is quasi-concave with respect to \(\omega_0\), where \(\Lambda^m(m_i(\omega), \omega, n) = v^d_i(m(\omega), \omega, n) - [(1 - F(\omega))/f(\omega)] \partial v^d_i(m(\omega), \omega, n)/\partial \omega\) the associated virtual surplus function and \(m_i(\omega)\) denotes the optimal allocation given \(\omega_0\).

**Proposition 10:** Suppose \((t_1^m, t_2^m)\) is a regular solution of the multi-platform monopoly problem. Let \(n_1(\omega)\) and \(n_2(\omega)\) be the induced advertising levels. Then there is a regular equilibrium \((t_1^d, t_2^d)\) of the corresponding duopoly game that induces the same advertising levels.

**Proof:** See Appendix 11.1.

### 10 Conclusion

This paper presented a platform competition model with either/both competition on the viewer side, allowing for fairly general viewer demand and advertising technologies. The generality of this framework allows the model to serve as a useful building block to tackle a variety of questions. For example, we took the quality of platforms exogenous in our analysis, yet competition in media markets (and in many other industries) often works through quality. Our model can be used to investigate whether markets in which users can be active on multiple platforms lead to higher or lower quality than those in which users are primarily active on a single platform. Another interesting question pertains to pricing tools. We considered the case in which platforms offer contracts consisting of an advertising level and a transfer, but in some industries firm primarily charge linear prices. How then do our results depend on the contracting

\[[(1 - F(\omega))/f(\omega)] \partial v^d_i(m_1, \omega, n)/\partial \omega.\] Again, our assumptions on viewer demand and on the advertising technology ensure strict quasi-concavity in \(m\) and the monotone hazard rate property ensures increasing differences in \((m, \omega)\) for values of \(m\) that are not too large.
environment? Also, do linear prices lead to a more or less competitive outcome? We leave these questions for future research.

Our model is also not restricted to the media markets context. In particular, a characterizing feature of our model is that consumers are multi-stop shoppers, i.e., can patronize multiple firms, but that a firms revenue is lower for a consumer, who buys from several other firms. The model is a first pass at understanding competition in settings where firms care not only about the overall demand but also about its composition. Such settings arise naturally when serving different types of customers yield different (indirect) revenues from other sources (as in our model), as well as when there are consumption externalities among customers.
References


11 Appendix

11.1 Proofs of Propositions

Proof of Proposition 2:

We first look at the last three terms in $W$, i.e., $\omega D_1 \phi_1 + \omega D_2 \phi_2 + \omega D_{12} \phi_{12}$. Taking the derivative of these terms gives (with arguments omitted):

$$\frac{\partial D_i}{\partial n_i} \phi_i + D_i \frac{\partial \phi_i}{\partial n_i} + \frac{\partial D_j}{\partial n_i} \phi_j + \frac{\partial D_{12}}{\partial n_i} \phi_{12} + D_{12} \frac{\partial \phi_{12}}{\partial n_i}.$$  \hspace{1cm} (13)

We can now substitute $\frac{\partial D_{12}}{\partial n_i} = -\frac{\partial D_j}{\partial n_i}$ into (13) to obtain

$$\frac{\partial D_i}{\partial n_i} \phi_i + D_i \phi_i' + \frac{\partial D_{12}}{\partial n_i} (\phi_{12} - \phi_j) + D_{12} \frac{\partial \phi_{12}}{\partial n_i}.$$ 

From (2) we know that at $n_i = n_i^d$ the last expression equals zero. However, the first terms in $W$ are the utilities of the viewers which are strictly decreasing in $n_i$. As a consequence, the first-order condition of $W$ with respect to $n_i$ evaluated at $n_i = n_i^d$ is strictly negative, which implies that there is too much advertising. ■

Proof of Proposition 3:

We know that the advertising level in case of duopoly is given by (6), while the advertising level of a single platform monopolist is given by (7). To check if advertising levels rise with entry, let us evaluate (6) at $n_i^{s.m.}$ and $n_i^{d}$. Since the first terms in both equations are the same, we get $n_i^d > n_i^{s.m.}$ if and only if

$$\frac{\partial D_{12}}{\partial n_i} (\phi_{12} - \phi_i - \phi_j) + D_{12} \left( \frac{\partial \phi_{12}}{\partial n_i} - \frac{\partial \phi_i}{\partial n_i} \right) > 0.$$ 

This is because, due to the fact that the objective functions are single-peaked, it follows that if the marginal revenue evaluated at the pre-entry advertising level is positive, given that platform $j$ sets $n_j^d$, then the incumbent’s equilibrium advertising level in duopoly must be larger. Rearranging this inequality gives (acknowledging the fact that $\phi_{12} - \phi_i - \phi_j < 0$)

$$-\frac{\partial D_{12}}{\partial n_i} D_{12} n_i > \left( \frac{\partial \phi_{12}}{\partial n_i} - \frac{\partial \phi_i}{\partial n_i} \right) \frac{n_i}{\phi_{12} - \phi_i - \phi_j}.$$ 

Since $\frac{\partial \phi_j}{\partial n_i} = 0$, we can write this inequality as

$$-\frac{\partial D_{12}}{\partial n_i} D_{12} n_i > \frac{\partial (\phi_i + \phi_j - \phi_{12})}{\partial n_i} \frac{n_i}{\phi_i + \phi_j - \phi_{12}}.$$ \hspace{1cm} (14)

Using our definitions

$$\eta_{D_{12}} := \frac{\partial D_{12}}{\partial n_i} D_{12} n_i$$

and

$$\eta_{\phi_i + \phi_j - \phi_{12}} := \frac{\partial (\phi_i + \phi_j - \phi_{12})}{\partial n_i} \frac{n_i}{\phi_i + \phi_j - \phi_{12}},$$
we can rewrite (14) as \( \eta_{D_{12}} > \eta_{\phi_i} + \phi_j - \phi_{12} \). Dividing this expression by \( \eta_{d_i} > 0 \), we obtain \( \eta_{D_{12}}/\eta_{d_i} > \eta_{\phi_i} + \phi_j - \phi_{12}/\eta_{d_i} \). Finally, note that from (7) we have \( \eta_{d_i} = \eta_{\phi_i} \), which yields

\[
\frac{\eta_{D_{12}}}{\eta_{d_i}} > \frac{\eta_{\phi_i} + \phi_j - \phi_{12}}{\eta_{\phi_i}}.
\]

\[\blacksquare\]

**Proof of Proposition 4:**

To show the result we look at the first-order condition

\[
\omega \left( \frac{\partial D_i}{\partial n_i} \phi_i + D_i \frac{\partial \phi_i}{\partial n_i} + \frac{\partial D_{12}}{\partial n_i} (\phi_{12} - \phi_j) + D_{12} \frac{\partial \phi_{12}}{\partial n_i} \right) = 0, \quad i, j = 1, 2; \ j = 3 - i.
\]

(15)

It is obvious that an increase in correlation leads to a fall of the second term since \( D_i \) decreases but to a rise of the fourth term since \( D_{12} \) increases. Since the overall demand \( D_i + D_{12} \) stays the same, the absolute changes in \( D_i \) and in \( D_{12} \) are the same. However, since the advertising technology is concave, implying that \( \partial \phi_i/\partial n_i > \partial \phi_{12}/\partial n_i \), in sum the change in the second and fourth terms diminish the left-hand side of (2). By our working assumption \( \partial^2 D_i/\partial n_i^2 = 0 \) and \( \partial^2 D_{12}/\partial n_i^2 = 0 \), the first and third term are unchanged. This implies that the first-order condition at the old advertising level is negative. Since there is a unique solution to (2), the new solution for \( n_i \) must be below the old solution. \[\blacksquare\]

**Proof of Proposition 6:**

We start with the case of duopoly, i.e., both platforms are in the market. First, look at the case of positive correlation. As can be seen from left diagram of Figure 2, the demand functions for the types that are uniformly distributed on the unit square are given by \( D_1 = (1 - n_1)n_2 \), \( D_2 = (1 - n_2)n_1 \) and \( D_{12} = (1 - n_1)(1 - n_2) \). For the types located on the 45-degree line, the demands are given by \( D_1 = \max\{n_2 - n_1, 0\} \), \( D_2 = \max\{n_1 - n_2, 0\} \) and \( D_{12} = 1 - \max\{n_1, n_2\} \).

![Figure 2: Positive correlation](image)

As evident in the left diagram of Figure 2, at \( n_1 = n_2 \) the demand function exhibits a kink. This is the case because at \( n_1 = n_2 \), \( D_1 = D_2 = 0 \) for the \( \lambda \)-types but \( D_i \) becomes positive if platform \( i \) reduces \( n_i \) slightly. Since there is a positive mass of \( \lambda \)-types, demand is kinked at this point.

To avoid this problem and to be able to use differentiation techniques, we perturb the model by
assuming that the $\lambda$-types are not just distributed on the 45-degree line but on the area that includes the space in $\epsilon$-distance around the 45-degree line. We will later let $\epsilon$ go to zero. This preference configuration with the $\epsilon$-area is displayed in the middle diagram of Figure 2. The advantage of this formulation is that, as shown in the right diagram of Figure 2, both $D_1$ and $D_2$ for the $\lambda$-types are strictly positive at $n_1 = n_2$. Therefore, when slightly changing $n_i$ around a symmetric equilibrium, the profit function $\Pi_i$ changes continuously, allowing us to apply differentiation techniques. After letting $\epsilon \to 0$, we obtain the equilibrium that arises when approaching the framework with viewers distributed on the 45-degree line.

We can now derive the demand functions for the viewers located on different points on the unit square. In the following we denote the demands by viewers in the $\epsilon$-area by $D_1^\epsilon$, $D_2^\epsilon$ and $D_{12}^\epsilon$ and the demands by the viewers outside this area by $D_1^s$, $D_2^s$ and $D_{12}^s$. This is illustrated in Figure 3.\(^{30}\)

![Figure 3: Demand areas with positive correlation](image)

Calculating the size of the $\epsilon$-area we obtain $2\epsilon(\sqrt{2} - \epsilon)$. Then determining the demands $D_1^\epsilon$ and $D_2^\epsilon$, we obtain from Figure 3 that they are given by the triangulars starting at the intersection point between the lines representing $n_1$ and $n_2$ and the lines confining the $\epsilon$-area. We can calculate the normalized demands, i.e., the demands such that the mass of viewers within and outside the $\epsilon$ area equals 1, so that $\lambda$ expresses the overall mass of the $\epsilon$-area and $1 - \lambda$ the mass outside this area. Calculating the respective demands gives

$$D_1^\epsilon = \frac{(\sqrt{2}\epsilon + n_2 - n_1)^2}{4\epsilon(\sqrt{2} - \epsilon)} \quad \text{and} \quad D_2^\epsilon = \frac{(\sqrt{2}\epsilon - n_2 + n_1)^2}{4\epsilon(\sqrt{2} - \epsilon)}.$$

\(^{30}D_{12}^s\) shows up twice just to express that both areas belong to $D_{12}^s$.\[^{29}\]
From that we can easily deduce
\[
D_1^s = \frac{2(1 - n_1)n_2 - (\sqrt{2}e + n_2 - n_1)^2}{2(1 - 2\epsilon(\sqrt{2} - e))}
\quad \text{and} \quad
D_2^s = \frac{2(1 - n_2)n_1 - (\sqrt{2}e - n_2 + n_1)^2}{2(1 - 2\epsilon(\sqrt{2} - e))}.
\]

Similarly, determining the demands for multi-homing viewers, we obtain
\[
D_{12}^s = \frac{(1 - n_2 - \sqrt{2}e)^2 + \frac{1}{2} (1 - n_1 - \sqrt{2}e)^2}{2(1 - 2\epsilon(\sqrt{2} - e))},
\]

implying that
\[
D_{12}^s = \frac{2(1 - n_1)(1 - n_2) - (1 - n_2 - \sqrt{2}e)^2 - (1 - n_1 - \sqrt{2}e)^2}{4\epsilon(\sqrt{2} - e)}.
\]

The profit function of platform \(i\) in duopoly is then given by
\[
\Pi_i^d = \omega \left[ (\lambda D_i^s + (1 - \lambda) D_1^s)(1 - e^{-n_i}) + (\lambda D_{12}^s + (1 - \lambda) D_{12}^s)(e^{-n_{12}} - e^{-(n_1 + n_2)}) \right] \quad (16)
\]

Differentiating leads to a first-order condition of
\[
\frac{\partial \Pi_i^d}{\partial n_1} = \left( \lambda \frac{\partial D_i}{\partial n_1} + (1 - \lambda) \frac{\partial D_1}{\partial n_1} \right) (1 - e^{-n_i}) + (\lambda D_i + (1 - \lambda) D_1)e^{-n_i}
\]
\[
+ \left( \lambda \frac{\partial D_{12}}{\partial n_1} + (1 - \lambda) \frac{\partial D_{12}}{\partial n_1} \right) (e^{-n_{12}} - e^{-(n_1 + n_2)}) + (\lambda D_{12} + (1 - \lambda) D_{12})e^{-(n_1 + n_2)} = 0, \quad (17)
\]
where the partial derivatives of the different demand regions with respect to \(n_i\) can be easily calculated from the demands given above.

Using that at a symmetric equilibrium \(n_1 = n_2 = n^*\) and letting \(\epsilon \to 0\), we obtain that \(n^*\) is implicitly given by
\[
\lambda n^* - n^* - \frac{\lambda}{2} + e^{-n^*} \left[ \lambda + 3n^* + \lambda (n^*)^2 - 1 - (n^*)^2 - 3\lambda n^* \right]
\]
\[
+ e^{-2n^*} \left[ 2 + (n^*)^2 + 2\lambda n^* - \frac{\lambda}{2} - 3n^* - \lambda (n^*)^2 \right] = 0. \quad (18)
\]

At \(\lambda = 0\), we obtain
\[
e^{-n^*} \left[ 3n^* - (n^*)^2 - 1 \right] + e^{-n^*} \left( 2 + (n^*)^2 - 3n^* \right) = n^*.
\]
Solving for \(n^*\) we obtain a unique solution given by \(n^* = 0.443\). At \(\lambda = 1\), (18) reduces to
\[
e^{-2n^*} \left( \frac{3}{2} - n^* \right) = \frac{1}{2}.
\]
Solving this yields \(n^* = 0.396\).

To determine how \(n^*\) changes with \(\lambda\) we can apply the Implicit Function Theorem to the first-order condition (17) and then evaluate it at a symmetric equilibrium \(n_1^* = n_2^*\). After letting \(\epsilon \to 0\) we obtain
\[
\text{sign} \left\{ \frac{dn^*}{d\lambda} \right\} = \text{sign} \left\{ -\frac{1}{2} + n^* - e^{-n^*} \left( 3n^* - 1 - (n^*)^2 \right) - e^{-2n^*} \left( \frac{1}{2} + (n^*)^2 - n^* \right) \right\}.
\]
It is easy to verify that for all values of $n^* \in [0.396, 0.443]$, $dn^*/d\lambda$ is strictly negative. But this implies that, in case of positive correlation, for all $\lambda \in [0, 1]$, $n^*$ strictly decreases with $\lambda$.

We now turn to the case of negative correlation. Here the analysis is simpler. However, we need to distinguish between the case in which $D_{12}^e > 0$ and the case in which $D_{12}^e = 0$. The first case is displayed on the left-hand side of Figure 4 and the second case on the right-hand side.

![Figure 4: Negative Correlation](image)

It is easy to see that in the first case the demand areas of the $\lambda$-types are given by

$$ D_1^e = n_2, \quad D_2^e = n_1, \quad \text{and} \quad D_{12}^e = (1 - n_1 - n_2), $$

while in the second case the demands areas are

$$ D_1^e = 1 - n_1, \quad D_2^e = 1 - n_2, \quad \text{and} \quad D_{12}^e = 0. $$

For the $(1 - \lambda)$-types in both cases we have

$$ D_1^s = (1 - n_1)n_2 \quad D_2^s = (1 - n_2)n_1 \quad D_{12}^s = (1 - n_1)(1 - n_2). $$

In the first case, we need to take into account that its demand configuration can only be an equilibrium if and only if $n_1 + n_2 \leq 1$ since otherwise $D_{12}^e = 0$. This implies that at a symmetric equilibrium $n^* \leq 0.5$. The profit functions and the first-order conditions can be written as in (16) and (17), just with the adapted demand function. We can again solve the first-order conditions for the symmetric equilibrium. Here we obtain that $n^*$ is defined by

$$ (\lambda - 1)n^* + e^{-n^*} \left[ 3n^* + (\lambda - 1)(n^*)^2 - 2\lambda n^* - 1 \right] + e^{-2n^*} \left[ 2 + \lambda n^* - 3n^* - (\lambda - 1)(n^*)^2 \right] = 0. $$

Applying the Implicit Function Theorem we get

$$ \text{sign} \left\{ \frac{dn^*}{d\lambda} \right\} = \text{sign} \left\{ n^* - e^{-n^*} n^* (2 - n^*) - e^{-2n^*} n^* (1 - n^*) \right\}. $$
which is positive for all \( n^* \in [0.443, 0.5] \). Inserting \( n^* = 0.5 \) into (19) and solving for \( \lambda \), we obtain \( \lambda = 0.529 \). Therefore, a symmetric equilibrium exists with the demand configuration given by the first case as long as \( \lambda \leq 0.529 \).

We can do the same analysis for the second case in which \( D_{12} = 0 \). However, deriving the first-order conditions for this case and solving for the symmetric equilibrium we obtain that \( n^* < 0.5 \) for all \( \lambda \in [0,1] \), implying that this demand configuration can never be an equilibrium.

Therefore, for \( \lambda > 0.529 \) the only symmetric equilibrium is that both platforms set \( n_i^* \) equal to 0.5, leaving \( D_{12} \) just equal to zero. Lowering the advertising level is not profitable since this does not lead to an increase in \( D_i^e \) because then the case \( D_i^e = n-i \) becomes relevant. However, increasing the advertising level is also not profitable since then \( D_i^e \) falls by too much due to the fact that the case \( D_i^e = 1-n_i \) is relevant. As a consequence, for negative correlation \( n^* \) is weakly increasing over \( \lambda \in [0,1] \); we obtain \( n^* = 0.443 \) at \( \lambda = 0 \), \( n_i^* \) strictly increases up to \( n^* = 0.5 \) at \( \lambda = 0.529 \) and stays at this level for \( \lambda \in [0.529,1] \).

We now turn to the single platform case. Keeping the demand notation as it was derived using Figure 3, the profit function of a monopolist owning a single platform can be written as

\[
\Pi_i^m = \omega \left[ (\lambda D_i^e + (1-\lambda)D_i^s + \lambda D_{12}^e + (1-\lambda)D_{12}^s) (1-e^{-n_i}) \right],
\]

which leads to the first-order condition

\[
\frac{\partial \Pi_i^m}{\partial n_i} = \left( \lambda \frac{\partial D_i^e}{\partial n_i} + (1-\lambda) \frac{\partial D_i^s}{\partial n_i} + \lambda \frac{\partial D_{12}^e}{\partial n_i} + (1-\lambda) \frac{\partial D_{12}^s}{\partial n_i} \right) (1-e^{-n_i})
\]

\[
+ (\lambda D_i^e + (1-\lambda)D_i^s + \lambda D_{12}^e + (1-\lambda)D_{12}^s) e^{-n_i} = 0.
\]

Inserting the respective values for the demands and the respective derivatives into this first-order condition and rearranging gives

\[
e^{-n_i^{s,m}} (2 - n_i^{s,m}) = 1.
\]

Therefore \( n_i^{s,m} \) is independent of \( \lambda \). Solving (20) for \( n_i^{s,m} \), we obtain \( n_i^{s,m} = 0.443 \). This corresponds to the equilibrium under duopoly for independent viewership. Since we know that \( n_i^* < 0.443 \) for positive correlation and \( n_i^* > 0.443 \) for negative correlation, the result of the proposition follows.

**Proof of Proposition 8:**

The equilibrium values of \( n_i \) and \( p_i \) are given by (10) and (11). Rewriting (11) yields

\[
p_i \left( \frac{\partial D_i}{\partial p_i} + \frac{\partial D_{12}}{\partial p_i} \right) = -\omega \left[ \frac{\partial D_i}{\partial p_i} \varphi_i + \frac{\partial D_{12}}{\partial p_i} (\varphi_{12} - \varphi_j) \right] - (D_i + D_{12}).
\]

To determine the relationship between \( \partial D_i/\partial n_i \) and \( \partial D_i/\partial p_i \), we can write \( D_i = \int_{\gamma_i + p_i}^{\infty} h(q_i, q_j) dq_j dq_i \) and \( D_{12} = \int_{\gamma_i + p_i}^{\infty} \int_{\gamma_i + p_i}^{\infty} h(q_i, q_j) dq_j dq_i \). This implies that

\[
\frac{\partial D_i}{\partial n_i} = -\gamma \int_0^{\gamma_i + p_j} h(\gamma_i + p_i, q_j) dq_j dq_i, \quad \frac{\partial D_i}{\partial p_i} = -\int_0^{\gamma_i + p_j} h(\gamma_n + p_i, q_j) dq_j dq_i,
\]

\[
\frac{\partial D_{12}}{\partial n_i} = -\gamma \int_{\gamma_i + p_i}^{\infty} h(\gamma_i + p_i, q_j) dq_j dq_i \quad \text{and} \quad \frac{\partial D_{12}}{\partial p_i} = -\int_{\gamma_i + p_i}^{\infty} h(\gamma_i + p_i, q_j) dq_j dq_i.
\]
Therefore, \( \partial D_i/\partial n_i = \gamma \partial D_i/\partial p_i \) and \( \partial D_{12}/\partial n_i = \gamma \partial D_{12}/\partial p_i \).

Inserting this into (21) and rearranging gives

\[
p_i \left( \frac{\partial D_i}{\partial n_i} + \frac{\partial D_{12}}{\partial n_i} \right) = -\omega \left[ \frac{\partial D_i}{\partial n_i} \phi_i + \frac{\partial D_{12}}{\partial n_i} (\phi_{12} - \phi_j) \right] - \gamma (D_i + D_{12}).
\] (22)

Substituting the last term of (10) into (22) and rearranging yields

\[
\omega \left[ D_i \frac{\partial \phi_i}{\partial p_i} + D_{12} \frac{\partial \phi_{12}}{\partial n_i} \right] - \gamma (D_i + D_{12}) = 0.
\] (23)

If the viewer preference correlation increases, then \( D_{12} \) increases and \( D_i \) falls by the same amount, given that advertising levels stay constant. Therefore, the second term of the left-hand side of (23) stays constant, while the first term falls since \( \partial \phi_i/\partial n_i > \partial \phi_i/\partial n_{12} \). As a consequence, the left-hand side of (23) is negative at the old solution (23). Since there is a unique solution to (23), the optimal value of \( n_i \) is falling with an increasing viewer preference correlation.

Now we turn to the optimal value of \( p_i \). If the second derivatives of the viewer pricing function are zero, i.e., \( \partial^2 D_i/\partial p_i^2 = 0 \) and \( \partial^2 D_{12}/\partial p_i^2 = 0 \), then a change in the viewer preference correlation does not change the first-order condition given by

\[
\omega \left[ \frac{\partial D_i}{\partial p_i} \phi_i + \frac{\partial D_{12}}{\partial p_i} (\phi_{12} - \phi_j) \right] + D_i + D_{12} + p_i \left( \frac{\partial D_i}{\partial p_i} + \frac{\partial D_{12}}{\partial p_i} \right) = 0,
\] (24)

if advertising levels stay constant. However, we know that advertising levels fall if correlation increases. This implies that \( \phi_i \) and \( \phi_{12} \) fall, while \( D_i + D_{12} \) rises. As a consequence, both \( \omega \left[ \partial D_i/\partial p_i \phi_i + \partial D_{12}/\partial p_i (\phi_{12} - \phi_j) \right] \) and \( D_i + D_{12} \) rise. Therefore, for the first-order condition to be satisfied, the last term must fall. Since \( \partial D_i/\partial p_i + \partial D_{12}/\partial p_i \) is negative, this implies that \( p_i \) must rise. \( \blacksquare \)

**Proof of Proposition 9:**

The profit function of a single platform monopolist in the case of viewer pricing is given by \( \omega \phi_i (D_i + D_{12}) + p_i (D_i + D_{12}) \). Hence, the optimal solutions \( n_i^{s,m} \) and \( p_i^{s,m} \) are characterized by the first-order conditions

\[
\omega \left[ \frac{\partial D_i}{\partial n_i} \phi_i + \frac{\partial D_{12}}{\partial n_i} (D_i + D_{12}) \right] + p_i \left( \frac{\partial D_i}{\partial n_i} + \frac{\partial D_{12}}{\partial n_i} \right) = 0
\] (25)

and

\[
\omega \phi_i \left( \frac{\partial D_i}{\partial p_i} + \frac{\partial D_{12}}{\partial p_i} \right) + D_i + D_{12} + p_i \left( \frac{\partial D_i}{\partial p_i} + \frac{\partial D_{12}}{\partial p_i} \right) = 0.
\] (26)

Using \( \partial D_i/\partial p_i = (1/\gamma) \partial D_i/\partial n_i \) and \( \partial D_{12}/\partial p_i = (1/\gamma) \partial D_{12}/\partial n_i \), rearranging (26), and inserting it into (25) lets us rewrite (25) as

\[
\omega \left( \frac{\partial \phi_i}{\partial n_i} - \gamma \right) (D_i + D_{12}) = 0.
\] (27)

From (23) we know that in duopoly \( n_i^d \) is given by

\[
\omega \left( \frac{\partial \phi_i}{\partial n_i} - \gamma \right) D_i + \omega \left( \frac{\partial \phi_{12}}{\partial n_i} - \gamma \right) D_{12} = 0.
\] (28)

Since \( \partial \phi_i/\partial n_i > \partial \phi_i/\partial n_{12} \), the left-hand side of (28) is smaller than the left-hand side of (27) implying
that
\[ n_i^d < n_i^{s,m}. \]

Turning to the comparison of \( p_i^d \) with \( p_i^{s,m} \), we need to compare the left-hand side of (26) and (24). Suppose that \( p_i^d = p_i^{s,m} \) and that \( \partial^2 D_i/\partial p_i^2 = \partial^2 D_{i2}/\partial p_i^2 = 0 \). Then, the last terms in both equations are the same. Since \( \phi_i > \phi_{12} - \phi_j \) for any \( n_i \), the first term of (26) is larger than the first term of (24). Since \( n_i^d < n_i^{s,m} \) and \( D_i \) and \( D_{i2} \) are decreasing in \( n_i \), it follows that \( D_i + D_{i2} \) in (26) is larger than in (24). Hence, the left-hand side of (26) is strictly larger than the one of (24), thereby contradicting \( p_i^d = p_i^{s,m} \). It follows that at the left-hand side of (26) is strictly positive at \( p_i^{s,m} \), implying \( p_i^d > p_i^{s,m} \). ■

Proof of Proposition 10:

We start with the problem of a duopolist \( i \) who chooses a price schedule to maximize profits
\[ \int_{\omega} t_i(m_i(\omega), \omega) dF(\omega), \]
given its rival’s choice \( t_j(m_j) \). From the main text this problem can be rewritten as in (12). Denote by \( m_j^*(m, \omega) \) the quantity that type \( \omega \) optimally buys from platform \( j \) when buying quantity \( m \) from platform \( i \). Then, the net contracting surplus for type \( \omega \) is
\[
\begin{align*}
  v_i^d(m, \omega, n) &= \max_y \left[ \omega u(m, y, n) - t_j(y) \right] - \left( \max_{y'} \left[ \omega u(0, y', n) - t_j(y') \right] \right) \\
  &= \omega u(m, m_j^*(m, \omega), n) - t_j(m_j^*(m, \omega)) - \omega u(0, m_j^*(0, \omega), n) - t_j(m_j^*(0, \omega))
\end{align*}
\]

Incentive compatibility requires \( m_i(\omega) = \arg\max_m v_i^d(m, \omega, n) - t_i(m) \), which implies
\[
  v_i^d(m_i(\omega), \omega, n) - t_i(m_i(\omega)) = \max_{y, y', m} \left\{ \omega u(m, y, n) - t_j(y) - \left( \omega u(0, y', n) - t_j(y') \right) - t_i(m) \right\}
\]

By the envelope theorem the derivative of the above with respect to \( \omega \) is
\[
  u(m, m_j^*(n_i(\omega), \omega), n) - u(0, m_j^*(0, \omega), n)
\]

Since the above pins down the growth rate of the agent’s payoff, we have that \( \max_{\omega_0, m_i(\cdot)} \int_{\omega_0} t_i(\omega) \) subject to the first two constraints of (12) equals
\[
\begin{align*}
  \max_{m_i(\cdot), \omega_0} \int_{\omega_0} & \left\{ \omega u(m_i(\omega), m_j^*(m_i(\omega), \omega), n) - \omega u(0, m_j^*(0, \omega), n) - t_j(m_j^*(m_i(\omega), \omega)) + t_j(m_j^*(0, \omega)) \\
  & - \int_{\omega_0} \left[ \omega u(m, m_j^*(n_i(z), z), n) - \omega u(0, m_j^*(0, z), n) \right] d\omega \right\} dF(\omega) \\
  &= \max_{\omega_0, m_i(\cdot)} \int_{\omega_0} \left\{ v_i^d(m_i(\omega), \omega, n) - \int_{\omega_0} \left[ \omega u(m, m_j^*(n_i(z), z), n) - \omega u(0, m_j^*(0, z), n) \right] d\omega \right\} dF(\omega)
\end{align*}
\]

Integrating the double integral by parts gives
\[
\begin{align*}
  \max_{m_i(\cdot), \omega_0} \int_{\omega_0} \omega u(m_i(\omega), m_j^*(m_i(\omega), \omega), n) - \omega u(0, m_j^*(0, \omega), n) - t_j(m_j^*(m_i(\omega), \omega)) + t_j(m_j^*(0, \omega)) + \\
  & - \frac{1 - F(\omega)}{f(\omega)} (u(m, m_j^*(n_i(\omega), \omega), n) - u(0, m_j^*(0, \omega), n)) dF(\omega)
\end{align*}
\]
The duopolist’s best reply allocation \( m_i^d(\omega) \) then solves

\[
\max_{m_i(\cdot),\omega_0} \int_{\omega_0}^{\omega} \left( \omega - \frac{1 - F(\omega)}{f(\omega)} \right) (u(m_i(\omega), m_j^*(m_i(\omega), \omega), n) - u(0, m_j^*(0, \omega), n)) - (t_j(m_j^*(m_i(\omega), \omega)) - t_j(m_j^*(0, \omega))) \, dF(\omega),
\]

subject to \( \int_{\omega}^{\omega'} m_i(\omega')dF(\omega') \leq n_i. \)

From now on we will denote the integrand function by \( \Lambda^d(m_i(\omega), \omega, n) \). Recall that solving a canonical screening problem usually involves maximizing the integral over all types served of the “full utility” of type \( \omega \) minus its informational rent, expressed as a function of the allocation. The “full utility” here is the incremental value \( u(m_i(\omega), m_j^*(m_i(\omega), \omega), n) - u(0, m_j^*(0, \omega), n), \) minus the difference in transfers.

Now consider the monopolist’s problem, which is to choose a pair of price schedules and a participation fee \( t_0 \leq \bar{t} < +\infty \), where \( \bar{t} \) is arbitrarily large. Without loss of generality, we restrict \( t_j(0), t_i(0) \leq 0 \). Analogous to the duopoly case, this is due to the fact that, conditional on paying the participation fee, all advertisers can guarantee a zero allocation at zero price at either platform. In the following, we define \( \tilde{t}_i(m_i(\omega)) := t_i(m_i(\omega)) + \bar{t}_i \), where \( \bar{t}_i \) is a constant to be determined by the monopolist. Given \( t_j(\cdot) \) the monopolist’s problem is

\[
\max_{t_i(\cdot),t_0,\bar{t},t_j} \int_{\omega_0}^{\omega} (\tilde{t}_i(m_i(\omega)) + \tilde{t}_j(m_j(\omega)) + t_0)I(m_i(\omega) + m_j(\omega) > 0) \, dF(\omega),
\]

where \( I \) is an indicator function equal to 1 whenever its argument is true and zero otherwise. The net contracting surplus corresponding to type \( \omega \) as a function of the allocation is

\[
v_i^m(m, \omega, n) = \max_y \left[ \omega u(m, y, n) - t_j(y) - \bar{t}_j - t_0 \right] - \sup \left\{ \max_y \left[ \omega u(0, y', n) - t_j(y') - \bar{t}_j - t_0 \right], 0 \right\}.
\]

As in the previous case, the problem given by (31) can be rewritten as a standard incentive problem of the form

\[
\max_{t_i(\cdot),m_i(\cdot),t_0,\bar{t},t_j} \int_{\omega_0}^{\omega} (\tilde{t}_i(m_i(\omega)) + \tilde{t}_j(m_j(\omega)) + t_0)I(m_i(\omega) + m_j(\omega) > 0) \, dF(\omega),
\]

subject to \( m_i(\omega) = \arg \max_m v_i^m(m, \omega, n) \) (incentive compatibility), \( v_i^m(m, \omega, n) - t_i(n_i(\omega)) - \bar{t}_i \geq 0 \) for all \( \omega \geq \omega_0 \) (individual rationality), and \( \int_{\omega_0}^{\omega} m_i(\omega')dF(\omega') \leq n_i \) (capacity constraint). By the envelope theorem the derivative of \( v_i^m(m_i(\omega), \omega, n) \) with respect to \( \omega \) is

\[
u(m_i(\omega), m_j^*(m_i(\omega), \omega, n)) - I(\omega, t_0)u(0, m_j^*(0, \omega), n),
\]

where \( I(\omega, t_0) \) is an indicator function equal to 1 if \( \max_{y'} \omega u(0, y', n) - t_j(y') - t_0 > 0 \) and zero otherwise. This coupled with individual rationality implies

\[
t_i(m_i(\omega)) = v_i^m(m, \omega, n) - \int_{\omega_0}^{\omega} \left( u(m_i(z), m_j^*(n_i(z), z), n) - \sup \{ u(0, n_j^*(0, z), n) \} \right) \, dz.
\]
Plugging this into the objective function we obtain

\[
\max_{m_i(\cdot), \omega, t_0, t_1, t_2} \int_{\omega_0}^{\infty} \left\{ \max_y \omega u(m_i(\omega), y, n) - \sup_{y'} \left\{ \max_{\omega'} \omega u(0, y', n) - t_j(y') - \bar{t}_j - t_0, 0 \right\} \right\} \\
- \int_{\omega_0}^{\omega} \left( u(m_i(z), m^*_j(m_i(z), n)) - I(\omega, t_0)u(0, m^*_j(0, z), n) \right) dz dF(\omega).
\]

(33)

Since \( \bar{t} \) is arbitrarily large and \( t_0 \) can be as large as \( \bar{t} \), there exists a \( t_0 \) such that \( t_0 > |\bar{t}_j| \). This implies that for \( t_0 \) large enough, \( \sup \left\{ \max_{\omega'} \omega u(0, y', n) - t_j(y') - \bar{t}_j - t_0, 0 \right\} = 0 \) and \( I(\omega, t_0) = 0 \). In addition, (33) is increasing in \( t_0 \). Hence, \( t_0 = \bar{t} \) and the monopolist’s problem (ignoring constraints) reduces to

\[
\max_{m_i(\cdot), \omega_0} \int_{\omega_0}^{\infty} \left\{ \max_y \omega u(m_i(\omega), y, n) - \int_{\omega_0}^{\omega} u(m_i(z), m^*_j(m_i(z), n))dz \right\} dF(\omega)
\]

Using the same technique as in the duopoly case, this gives

\[
\max_{m_i(\cdot), \omega_0} \int_{\omega_0}^{\infty} \left( \omega - \frac{1 - F(\omega)}{f(\omega)} \right) u(m_i(\omega), m^*_j(m_i(\omega), n))dF(\omega)
\]

subject to \( \int_{\omega_0}^{\omega} m_i(\omega')dF(\omega') \leq n_t \).

(34)

The above integrand, labeled \( \Lambda^m(m_i(\omega), \omega, n) \), reflects the “full surplus” internalization feature of our monopolist, similar to the homogeneous case. Here transfers do not show up because advertisers do not have the option to buy only one contract.

By our regularity assumptions, a solution exists to both problems: \((m^m_i(\omega), \omega^m_0), (m^d_i(\omega), \omega^d_0)\). To show that the allocation in both problems is the same, we will first establish that the optimal \( m_i(\omega) \) equals the \( \arg \max_q \) of \( \Lambda^d(q, \omega, n) \) and of \( \Lambda^m(q, \omega, n) \) and that the indifferent advertiser \( \omega_0 \) is also the same. We then turn to the optimal \( n_t \).

Let us first consider the schedule that fixes the marginal advertiser in both problems, and assume \( \omega^m_0 = \omega^d_0 \). The only difference between monopoly and duopoly is that in duopoly there is an additional term \( t^*_j(m^*_j(m_i(\omega), \omega), \omega) \), that depends on \( m_i \). However, applying the envelope theorem, it is evident from the definition of \( v^d_i(m, \omega, n) \) given in (29) that, when differentiating the integrand of the duopolist’s problem given by (30) with respect to \( m_i \), we can ignore the (indirect) effect of \( m_i \) on \( m^*_j \). The same argument applies to the monopolist’s problem given by (34), as can be seen from \( v^m_i(m, \omega, n) \) in (32). Therefore, the optimal solution for a duopolist and for a monopolist are the same.

With \( \omega^m_0 = \omega^d_0 \), we have established the following:

\[
m^m_i(\omega) = \begin{cases} 
  m^d_i(\omega) & \omega \geq \omega^m_0 \\
  0 & \text{otherwise}
\end{cases}
\]

This result implies that neutrality carries over on the “intensive” margin. That is, conditional on type \( \omega \) getting some positive allocation, both a monopolist and a duopolist best respond to some \( t_j \) by offering the same allocation. This is true because the maximization problems with respect to \( m_i(\cdot) \) are equivalent for a monopolist and duopolist, if \( w^m_0 = w^d_0 \).

We now turn to the extensive margin and will establish that \( \omega^m_0 = \omega^d_0 \). First, note that \( \Lambda^d = 0 \) at \( m_i = 0 \) for all \( \omega \). The increasing differences property \( \Lambda^d_{m_i, \omega} \geq 0 \) implies that the optimal allocation
is weakly monotone.\textsuperscript{31} As a consequence, the marginal type is defined as the highest type for which \( m_i(\omega) = 0 \). Therefore, we have \( m_i^d(\omega) = 0 \) for all \( \omega \leq \omega_0^d \).

Further note \( \Lambda^d(m_i^d(\omega), \omega, n) \geq 0 \) because \( \Lambda^d(0, \omega, n) = 0 \) for all \( \omega \) is a lower bound on \( \Lambda^d(x, \omega, n) \), \( x \geq 0 \). By the definition of \( \omega_0^d \), in a right neighborhood \( m_i^d(\omega) > 0 \); therefore, \( u(m_i(\omega), m_j^*(m_i(\omega), n)) - u(0, m_j^*(0, \omega), n) > 0 \) and \( t_j(m_j^*(m_i(\omega))) - t_j(m_j^*(0, \omega)) \geq 0 \). Hence, \( \Lambda^d(m_i(\omega), \omega, n) \geq 0 \) only if \( \omega - (1 - F(\omega))/f(\omega) \geq 0 \) in a right neighborhood of \( \omega_0^d \). By continuity and the monotone hazard rate property we have \( \omega - (1 - F(\omega))/f(\omega) \geq 0 \) for all \( \omega \geq \omega_0^d \). It follows that \( \Lambda^m(m_i^m(\omega), \omega, n) \geq 0 \) for all \( \omega \geq \omega_0^d \).

Now suppose that the monopolist would exclude the marginal type \( \omega \) for which \( \Lambda^m(m_i^m(\omega), \omega, n) \geq 0 \). This would entail a first-order loss but only a second-order gain. This is because type \( \omega \) pays a (weakly) positive transfer (recall \( m_j(\omega) \geq 0 \) and therefore \( t_j(m_j(\omega)) \geq 0 \)) but \( m_i(\omega) \) is arbitrarily close to zero, so the gain for all other advertisers when excluding the marginal type becomes negligible. Therefore, it is a local maximum to serve the marginal type for whom \( \Lambda^m(m_i^m(\omega), \omega, n) \geq 0 \). But since the profit function is quasi-concave in \( \omega_0 \), this is also a global maximum. Hence, \( \omega_0^m \leq \omega_0^d \). This coupled with the fact that \( m_i^m(\omega) = m_i^d(\omega) \) implies that the marginal price schedules must coincide: \( t_i^m(m) = t_i^d(m) \). As a consequence, \( \omega_0^m = \omega_0^d \).

To determine the optimal \( n_i \), we compare the optimization problems in (30) and (34). Knowing that the optimal price schedule and the marginal type is the same, it is easy to see that the maximization problems with respect to \( n_i \) coincide. This is because the terms that distinguish the two maximization problems from each other are independent of \( n_i \). As a consequence, \( n_i^m = n_i^d \). \( \blacksquare \)

11.2 Proofs of Other Results

Two-stage game with posted contracts

Consider the following assumptions:

A1 Platforms are symmetric.

A2 For any \( \alpha \in [0, 1] \), the following inequality holds

\[
    t_i^*(1 - \alpha) > \alpha \left\{ d_i(\alpha n_i^d) \phi(n_i^d) - d_i((1 - \alpha) n_i^*\phi(n_i^*)) \right\},
\]

(35)

where \( d_i(\cdot) := D_i(\cdot) + D_{12}(\cdot) \), \( n_i^d = \arg\max_{n_i} d_i(\alpha n_i) \phi(n_i) \), \( n_i^* \) is implicitly defined by (2) and \( t_i^* \) is implicitly defined by (1) with \( n_i = n_i^* \) and \( n_j = n_j^* \).

We provide a discussion of these assumption after the proof of the following proposition. There we explain that A1 can be weakened while A2 is a relatively natural assumption in our framework.

**Proposition** Suppose that A1 and A2 hold. Then, there is an equilibrium in the two-stage game game with posted contracts, that is outcome-equivalent to the equilibrium of the game defined in Section 3.

\textsuperscript{31}Even without increasing differences, incentive compatibility would restrict us to optimize with respect to monotone \( m_i(\omega) \) only.
Proof:
Suppose that in the two-stage game with posted contracts each platform offers a contract with \( n_i = n_i^* \), where \( n_i^* \) is implicitly determined by (2), and a transfer

\[
t_i^* = D_i(n_i^*, n_j^*)\phi_i(n_i^*) + D_{12}(n_i^*, n_j^*) (\phi_{12}(n_i^*, n_j^*) - \phi_j(n_j^*)) .
\]

By the same argument as we used for the original model, these contracts will be accepted by all advertisers. As this is anticipated by viewers, viewerships are \( D_i(n_i^*, n_j^*) \) and \( D_{12}(n_i^*, n_j^*) \). Since advertising levels are the same as in the equilibrium of the original model, viewerships are also the same. Therefore, this candidate equilibrium is outcome-equivalent to the equilibrium of the original model.

Let us now consider if there exists a profitable deviation from this candidate equilibrium. We first show that there can be no profitable deviation contract of platform \( i \) that still induces full advertiser participation on platform \( j \) but a smaller participation on platform \( i \). Let \( x_i \) denote the fraction of advertisers who accept the offer of platform \( i \).

Consider a candidate contract \((n_i, t_i)\). Suppose that platform \( i \)'s equilibrium payoff from this contract is \( t_i x_i \). Now consider the following alternative contract: \((x_i n_i, x_i t_i)\). Note that total advertising on platform \( i \) is still equal to \( x_i n_i \). So platform \( i \) is at least as attractive as with the candidate equilibrium contract. Note moreover that because \( \phi_i \) and \( \phi_{12} \) are strictly concave in \( n_i \), the incremental value of accepting offer \((x_i n_i, x_i t_i)\) must exceed \( x_i t_i \) for all levels of advertiser participation. So all advertisers would accept \((x_i n_i, x_i t_i)\) regardless. It follows that platform \( i \) can marginally increase \( x_i t_i \) while still getting full participation and therefore profits would strictly increase. It follows that no offer inducing a level of participation \( x_i < 1 \) can be part of a best reply.

Now suppose platform \( i \) deviates from the candidate equilibrium in such a way that it induces a fraction \( \alpha \) of the advertisers to single-home on its platform while the remaining fraction \( 1 - \alpha \) single-homes on platform \( j \). Defining \( d_i(\cdot) := D_i(\cdot) + D_{12}(\cdot) \), the largest possible transfer that platform \( i \) can ask is then bounded above by

\[
t_i^d = d_i(\alpha n_i^d, (1 - \alpha)n_j^d\phi_i(n_i^d) - u_{shj},
\]

where \( n_i^d \) denotes the optimal deviation advertising level and \( u_{shj} \) denotes the payoff of an advertiser who chooses to reject the contract of platform \( i \) and instead single-homes on platform \( j \). To determine \( u_{shj} \) we determine the advertiser’s payoff when accepting only platform \( j \)'s contract, which is the platform’s equilibrium contract after platform \( i \) has deviated to induce a fraction \( \alpha \) of advertisers to single-home on platform \( i \). We obtain

\[
 u_{shj} = d_j((1 - \alpha)n_j^d, \alpha n_i^d\phi_j(n_j^d) - t_j^* =
 \]

\[
d_j((1 - \alpha)n_j^d, \alpha n_i^d\phi_j(n_j^d) - D_j(n_j^d, n_i^d)\phi_j(n_j^d) - D_{12}(n_i^d, n_j^d) (\phi_{12}(n_i^d, n_j^d) - \phi_i(n_i^d)) .
\]

Platform \( i \)'s profit is then \( \alpha t_i^d \). Hence, deviating is not profitable if

\[
 \alpha \left\{ d_i(\alpha n_i^d\phi_i(n_i^d) - d_j((1 - \alpha)n_j^d)\phi_j(n_j^d) + D_j(n_j^d, n_i^d)\phi_j(n_j^d) + D_{12}(n_i^d, n_j^d) (\phi_{12}(n_i^d, n_j^d) - \phi_i(n_i^d)) \right\}
 \]

\[
 < D_i(n_i^d, n_j^d)\phi_i(n_i^d) + D_{12}(n_i^d, n_j^d) (\phi_{12}(n_i^d, n_j^d) - \phi_j(n_j^d)) .
\]
Now suppose that the two platforms are symmetric. Then the above condition reduces to
\[ \alpha \left( d_i(n^d_i) \phi(n^d_i) - d_i((1-\alpha)n^* \phi(n^*)) \right) - (1-\alpha) \left( D_i(n^*, n^*) \phi(n^*) + D_{12}(n^*, n^*) (\phi_{12}(n^*, n^*) - \phi(n^*)) \right) < 0, \]
where \( n_i^* = n_j^* = n^* \), \( n_i^d = n^d \), and \( \phi_i(\cdot) = \phi_j(\cdot) = \phi(\cdot) \). This can be rewritten as
\[ t_i^*(1-\alpha) > \alpha \left( d_i(n^d_i) \phi(n^d_i) - d_i((1-\alpha)n^*_i \phi(n^*_i)) \right). \]
which is fulfilled by \( A2 \). As a consequence, a deviation is not profitable.

We now shortly explain why the assumptions \( A1 \) and \( A2 \) are not very restrictive in our framework. First, consider \( A1 \). Since the game is continuous, \( A1 \) can be relaxed to some extent without affecting the result, implying that the proposition still holds if platforms are not too asymmetric. Now consider \( A2 \). It is evident from (35), that the assumption is clearly fulfilled for \( \alpha \) low enough. In this case the right-hand side is close to 0, while the left-hand side is strictly positive. Now consider the opposite case, i.e., \( \alpha \to 1 \).

In that case the left-hand side goes to zero, while the right-hand side goes to \( d_i(n^d_i) \phi(n^d_i) - d_i(0) \phi(n^*_i) \).
Evidently, \( d_i(0) > d_i(n^d_i) \). Hence, the right-hand side is negative if \( \phi(n^*_i) \) is not much larger than \( \phi(n^d_i) \).
In general, \( n^*_i \) can be larger or smaller than \( n^d_i \), implying that the difference can be either way. However, even in case \( n^d_i > n^*_i \), if the slope of the advertising functions \( \phi_i \) and \( \phi_{12} \) is relatively small, we obtain that the difference between \( n^*_i \) and \( n^d_i \) is small and so the right-hand side is negative. Finally, consider intermediate values of \( \alpha \). Again, if the difference between \( n^*_i \) and \( n^d_i \) is relatively small, the term in the bracket on the right-hand side of (35) is close to zero. Since the left-hand side is strictly positive, \( A2 \) is then fulfilled as well.

**Proof that the neutrality result extends to \( N \) platforms and any ownership structure**

Before showing the result, we start with an example of three platforms, that conveys the arguments.

The viewer demand structure in the case with three platforms is

- Multi-homers123: \( D_{123} := \text{Prob} \{ q_1 - \gamma n_1 \geq 0; q_2 - \gamma n_2 \geq 0; q_3 - \gamma n_3 \geq 0 \} \),
- Multi-homers12: \( D_{12} := \text{Prob} \{ q_1 - \gamma n_1 \geq 0; q_2 - \gamma n_2 \geq 0; q_3 - \gamma n_3 < 0 \} \),
- Multi-homers13: \( D_{13} := \text{Prob} \{ q_1 - \gamma n_1 \geq 0; q_2 - \gamma n_2 < 0; q_3 - \gamma n_3 \geq 0 \} \),
- Multi-homers23: \( D_{23} := \text{Prob} \{ q_1 - \gamma n_1 < 0; q_2 - \gamma n_2 \geq 0; q_3 - \gamma n_3 \geq 0 \} \),
- Single-homers1: \( D_1 := \text{Prob} \{ q_1 - \gamma n_1 \geq 0; q_2 - \gamma n_2 < 0; q_3 - \gamma n_3 < 0 \} \),
- Single-homers2: \( D_2 := \text{Prob} \{ q_1 - \gamma n_1 < 0; q_2 - \gamma n_2 \geq 0; q_3 - \gamma n_3 \geq 0 \} \),
- Single-homers3: \( D_3 := \text{Prob} \{ q_1 - \gamma n_1 < 0; q_2 - \gamma n_2 < 0; q_3 - \gamma n_3 \geq 0 \} \),
- Zero-homers: \( D_0 : = 1 - D_1 - D_2 - D_{12} - D_{13} - D_{23} - D_{123} \).

Because of our independence assumption we have
\[ \frac{\partial D_{ij}}{\partial n_j} = -\frac{\partial D_i}{\partial n_j} \quad \text{and} \quad \frac{\partial D_{ijk}}{\partial n_k} = -\frac{\partial D_{ij}}{\partial n_k}, \quad i,j,k = 1,2,3, \quad i \neq j, k; j \neq k. \]

In the same way as in the proofs if Claims 1 and 2, we can show that it is optimal for a monopolist and for oligopolists to offer only one contract, and this contract is accepted by all advertisers. Also, \( m_i = n_i, i = 1,2,3 \), in both cases and the monopolist can extract the full surplus from advertisers while an oligopolist can only extract the incremental surplus.

If all three platforms are controlled by different owners (full oligopoly), platform \( i \)'s maximization
problem is then (normalizing $\omega$ to unity)

$$
\Pi_i = D_i(n_1, n_2, n_3)\phi_i(n_i) + D_{ij}(n_1, n_2, n_3) (\phi_{ij}(n_i, n_j) - \phi_j(n_j)) \\
+ D_{ik}(n_1, n_2, n_3) (\phi_{ik}(n_i, n_k) - \phi_k(n_k)) + D_{123}(n_1, n_2, n_3) (\phi_{123}(n_i, n_j, n_k) - \phi_{jk}(n_j, n_k)).
$$

Differentiating this profit function with respect to $n_i$ yields (dropping arguments for simplicity)

$$
\frac{\partial D_i}{\partial n_i} \phi_i + D_i \frac{\partial \phi_i}{\partial n_i} + \frac{\partial D_{ij}}{\partial n_i} (\phi_{ij} - \phi_j) + D_{ij} \frac{\partial \phi_{ij}}{\partial n_i}
$$

$$
+ \frac{\partial D_{ik}}{\partial n_i} (\phi_{ik} - \phi_k) + D_{ik} \frac{\partial \phi_{ik}}{\partial n_i} + \frac{\partial D_{123}}{\partial n_i} (\phi_{123} - \phi_{jk}) + D_{123} \frac{\partial \phi_{123}}{\partial n_i} = 0. 
$$

The problem of a monopolist is

$$
\Pi_i^m = D_1(n_1, n_2, n_3)\phi_1(n_1) + D_2(n_1, n_2, n_3)\phi_2(n_2) + D_3(n_1, n_2, n_3)\phi_3(n_3) + D_{12}(n_1, n_2, n_3)\phi_{12}(n_1, n_2)
$$

$$
+ D_{13}(n_1, n_2, n_3)\phi_{13}(n_1, n_3) + D_{23}(n_1, n_2, n_3)\phi_{23}(n_2, n_3) + D_{123}(n_1, n_2, n_3)\phi_{123}(n_1, n_2, n_3)
$$

leading to a first-order condition of

$$
\frac{\partial D_1}{\partial n_1} \phi_1 + D_1 \frac{\partial \phi_1}{\partial n_1} + \frac{\partial D_{12}}{\partial n_1} \phi_{12} + \frac{\partial D_{13}}{\partial n_1} \phi_{13} = 0.
$$

Now subtracting (36) from (37), assuming that $n_j$ and $n_k$ are the same under monopoly and oligopoly gives

$$
\frac{\partial D_j}{\partial n_i} \phi_j + \frac{\partial D_k}{\partial n_i} \phi_k + \frac{\partial D_{jk}}{\partial n_i} \phi_{jk} + \frac{\partial D_{ij}}{\partial n_i} \phi_{ij} + \frac{\partial D_{ik}}{\partial n_i} \phi_{ik} + \frac{\partial D_{123}}{\partial n_i} \phi_{123} = 0.
$$

But due to the independence of the demand system we know that

$$
\frac{\partial D_j}{\partial n_i} + \frac{\partial D_{ij}}{\partial n_i} = 0, \quad \frac{\partial D_k}{\partial n_i} + \frac{\partial D_{ik}}{\partial n_i} = 0, \quad \text{and} \quad \frac{\partial D_{jk}}{\partial n_i} + \frac{\partial D_{123}}{\partial n_i} = 0,
$$

which implies that (38) equals zero. As a consequence, $n_i$ in monopoly and oligopoly are the same, given that $n_j$ and $n_k$ are the same. Since the same analysis applies to $n_j$ and $n_k$ and since there is a unique solution, we have established the neutrality result.

We can now also establish neutrality for the case in which an owner controls more than one platform but not all of them. Suppose that an owner controls platforms $i$ and $j$. The profit function is

$$
\Pi_{ij}' = D_i(n_1, n_2, n_3)\phi_i(n_i) + D_{ij}(n_1, n_2, n_3)\phi_{ij}(n_i, n_j)
$$

$$
+ D_{ik}(n_1, n_2, n_3) (\phi_{ik}(n_i, n_k) - \phi_k(n_k)) + D_{jk}(n_1, n_2, n_3) (\phi_{jk}(n_i, n_k) - \phi_k(n_k))
$$

$$
+ D_{123}(n_1, n_2, n_3) (\phi_{123}(n_i, n_j, n_k) - \phi_k(n_k)).
$$

The first-order condition for $n_i$ is then

$$
\frac{\partial D_i}{\partial n_i} \phi_i + D_i \frac{\partial \phi_i}{\partial n_i} + \frac{\partial D_{ij}}{\partial n_i} \phi_{ij} + D_{ij} \frac{\partial \phi_{ij}}{\partial n_i} + \frac{\partial D_{ik}}{\partial n_i} \phi_{ik} + D_{ik} \frac{\partial \phi_{ik}}{\partial n_i} 
$$

$$
+ \frac{\partial D_{jk}}{\partial n_i} \phi_{jk} + D_{jk} \frac{\partial \phi_{jk}}{\partial n_i} + \frac{\partial D_{123}}{\partial n_i} \phi_{123} + D_{123} \frac{\partial \phi_{123}}{\partial n_i} = 0.
$$
viewer behavior can be written as
\[ \frac{\partial D_{123}}{\partial n_i} (\phi_{123} - \phi_k) + D_{123} \frac{\partial \phi_{123}}{\partial n_i} + \frac{\partial D_{123}}{\partial n_i} (\phi_{123} - \phi_k) = 0. \]
Subtracting (39) from (36) (assuming that \( n_j \) and \( n_k \) are the same in both problems) gives
\[ -\frac{\partial D_{123}}{\partial n_i} \phi_j - \frac{\partial D_{jk}}{\partial n_i} \phi_{jk} + \frac{\partial D_{jk}}{\partial n_i} \phi_k + \frac{\partial D_{123}}{\partial n_i} \phi_k. \]
But due to independence, \( \frac{\partial D_{123}}{\partial n_i} + \frac{\partial D_{jk}}{\partial n_i} = 0 \), and we again have neutrality.

We now turn to the case of \( N \) firms. With \( N \) firms the assumption of mutual independence of the viewer behavior can be written as
\[ \frac{\partial (D_J + D_{J/j})}{\partial n_i} = 0, \]
with
\[ J = \{1, 2, 3, ..., N, 12, 13, ..., 1N, 23, 24, ..., 2N, 34, ..., (N - 1)N, \]
\[ 123, 124, ..., 12N, 234, ..., (N - 2)(N - 1)N, ..., 123 ...(N - 1)N \}\]
and
\[ j = \{1, 2, 3, ..., N \}, \ j \neq i. \]
That is, \( J \) is the set of all subsets of \( N \) platforms and \( J/j \) is the same subset with one platform excluded where this platform is not platform \( i \). So in the example with three platforms \( J = \{1, 2, 3, 12, 13, 23, 123\} \).

Let us first look at the case in which all \( N \) platforms are controlled by different owners. The profit function of platform \( i \) is then
\[ \Pi_i^t = D_i \phi_i + \sum_{k = 1 \atop k \neq i}^{N} D_{ki} (\phi_{ki} - \phi_k) + \sum_{k_1, k_2 = 1 \atop k_1, k_2 \neq i}^{N} D_{k1k2} (\phi_{k1k2i} - \phi_{k1k2}) \]
\[ + \sum_{k_1, k_2, k_3 = 1 \atop k_1, k_2, k_3 \neq i \atop k_3 > k_2 > k_1}^{N} D_{k1k2k3} (\phi_{k1k2k3i} - \phi_{k1k2k3}) + ... + D_{12...i...N} (\phi_{12...i...N} - \phi_{12...i...1i+1,...N}). \]
Here the argument of any viewer demand function \( D_k \) is \( n_1, n_2, ..., n_i, ..., n_N \), and the argument of the \( \phi_k \)-functions consists of the respective advertising levels, e.g., \( \phi_{k1k2i} = \phi_{k1k2i}(n_{k1}, n_{k2}, n_i) \).

The first-order condition with respect to \( n_i \) is
\[ \frac{\partial D_i}{\partial n_i} \phi_i + D_i \frac{\partial \phi_i}{\partial n_i} + \sum_{k = 1 \atop k \neq i}^{N} \left[ \frac{\partial D_{ki}}{\partial n_i} (\phi_{ki} - \phi_k) + D_{ki} \frac{\partial \phi_{ki}}{\partial n_i} \right] + \]
\[ + \sum_{k_1, k_2 = 1 \atop k_1, k_2 \neq i \atop k_2 > k_1}^{N} \left[ \frac{\partial D_{k1k2i}}{\partial n_i} (\phi_{k1k2i} - \phi_{k1k2}) + D_{k1k2i} \frac{\partial \phi_{k1k2i}}{\partial n_i} \right] \]
\[ (40) \]
Differentiating with respect to $n_i$

$$
+ \sum_{k_1, k_2, k_3 = 1}^{N} \left[ \frac{\partial D_{k_1 k_2 k_3 i}}{\partial n_i} (\phi_{k_1 k_2 k_3 i} - \phi_{k_1 k_2 k_3}) + D_{k_1 k_2 k_3 i} \frac{\partial \phi_{k_1 k_2 k_3 i}}{\partial n_i} \right]
$$

$$
+ \ldots + \frac{\partial D_{i+1 \ldots N}}{\partial n_i} (\phi_{i+1 \ldots N} - \phi_{i-1 \ldots i+1 \ldots N}) + D_{i+1 \ldots N} \frac{\partial \phi_{i+1 \ldots N}}{\partial n_i} = 0.
$$

Now we turn to the problem of a monopolist with profit function given by

$$
\Pi_M = \sum_{j=1}^{N} \left\{ D_j \phi_j + \sum_{k=1}^{N} D_{kj} \phi_{kj} + \sum_{k_1, k_2, k_3 = 1}^{N} D_{k_1 k_2 j} \phi_{k_1 k_2 j} + \sum_{k_1, k_2, k_3 = 1}^{N} D_{k_1 k_2 k_3 j} \phi_{k_1 k_2 k_3 j} + \ldots \right\}
$$

Differentiating with respect to $n_i$ gives

$$
\frac{\partial D_i}{\partial n_i} \phi_i + D_i \frac{\partial \phi_i}{\partial n_i} + \sum_{j=1}^{N} \frac{\partial D_j}{\partial n_i} \phi_j + \sum_{k=1, j \neq i}^{N} \left[ \frac{\partial D_{ki}}{\partial n_i} \phi_{ki} + D_{ki} \frac{\partial \phi_{ki}}{\partial n_i} \right] + \sum_{j=1, k \neq i}^{N} \sum_{k=1, j \neq i}^{N} \frac{\partial D_{kj}}{\partial n_i} \phi_{kj}
$$

$$
+ \sum_{k_1, k_2, k_3 = 1}^{N} \left[ \frac{\partial D_{k_1 k_2 j}}{\partial n_i} \phi_{k_1 k_2 j} + D_{k_1 k_2 j} \frac{\partial \phi_{k_1 k_2 j}}{\partial n_i} \right] + \sum_{j=1, k \neq i}^{N} \sum_{k_1, k_2, k_3 = 1}^{N} \left[ \frac{\partial D_{k_1 k_2 k_3 j}}{\partial n_i} \phi_{k_1 k_2 k_3 j} + D_{k_1 k_2 k_3 j} \frac{\partial \phi_{k_1 k_2 k_3 j}}{\partial n_i} \right]
$$

$$
+ \sum_{j=1, k \neq i}^{N} \sum_{k_1, k_2, k_3 = 1}^{N} \frac{\partial D_{k_1 k_2 k_3 j}}{\partial n_i} \phi_{k_1 k_2 k_3 j} + \ldots + \frac{\partial D_{i+1 \ldots N}}{\partial n_i} \phi_{i+1 \ldots i \ldots N} + D_{i+1 \ldots N} \frac{\partial \phi_{i+1 \ldots i \ldots N}}{\partial n_i} = 0.
$$

Now we can take the difference between the left-hand sides of (41) and (40), given that all $n_j$, $j = 1, \ldots, N$, $j \neq i$ are equal in monopoly and oligopoly:

$$
\sum_{k=1}^{N} \phi_k \left( \frac{\partial D_k}{\partial n_i} + \frac{\partial D_{ki}}{\partial n_i} \right)
$$
Rewriting the third and the fifth term to the same arguments as before and is therefore omitted.

\[ + \sum_{j=1}^{N} \sum_{\substack{k=1 \\ j \neq i \ \ k \neq i \ \ k2 \neq j}}^{N} \frac{\partial D_{kj}}{\partial n_i} \phi_{kj} + \sum_{\substack{k1, k2 = 1 \\ k1, k2 \neq i \ \ k2 > k1}}^{N} \frac{\partial D_{k1k2i}}{\partial n_i} \phi_{k1k2} \]

\[ + \sum_{j=1}^{N} \sum_{\substack{k1, k2 = 1 \\ k1, k2 < j \ \ k1, k2 \neq i \ \ k2 > k1}}^{N} \frac{\partial D_{k1k2j}}{\partial n_i} \phi_{k1k2j} + \sum_{\substack{k1, k2, k3 = 1, \\ k1, k2, k3 \neq i \ \ k2 > k1 \ \ k3 > k2}}^{N} \frac{\partial D_{k1k2k3i}}{\partial n_i} \phi_{k1k2k3} + ... + \]

\[ + \frac{\partial D_{12...i-1i+1...N}}{\partial n_i} \phi_{12...i-1i+1...N} + \frac{\partial D_{12...i...N}}{\partial n_i} \phi_{12...i-1i+1...N} \]

Rewriting the third and the fifth term to

\[ + \sum_{j=1}^{N} \sum_{\substack{k=1 \\ j \neq i \ \ k \neq i \ \ k2 > k1}}^{N} \frac{\partial D_{kj}}{\partial n_i} \phi_{kj} = \sum_{\substack{k1, k2 = 1 \\ k1, k2 \neq i \ \ k2 > k1}}^{N} \frac{\partial D_{k1k2i}}{\partial n_i} \phi_{k1k2} \]

and

\[ \sum_{j=1}^{N} \sum_{\substack{k1, k2 = 1 \\ k1, k2 < j \ \ k1, k2 \neq i \ \ k2 > k1}}^{N} \frac{\partial D_{k1k2j}}{\partial n_i} \phi_{k1k2j} = \sum_{\substack{k1, k2, k3 = 1, \ \ k1, k2, k3 \neq i \ \ k2 > k1 \ \ k3 > k2}}^{N} \frac{\partial D_{k1k2k3i}}{\partial n_i} \phi_{k1k2k3} \]

allows to write (42) as

\[ \sum_{k=1}^{N} \phi_k \left( \frac{\partial D_k}{\partial n_i} + \frac{\partial D_{ki}}{\partial n_i} \right) + \sum_{\substack{k1, k2 = 1 \\ k1, k2 \neq i \ \ k2 > k1}}^{N} \phi_{k1k2} \left( \frac{\partial D_{k1k2i}}{\partial n_i} + \frac{\partial D_{k1k2i}}{\partial n_i} \right) \]

\[ + \sum_{\substack{k1, k2, k3 = 1, \ \ k1, k2, k3 \neq i \ \ k2 > k1 \ \ k3 > k2}}^{N} \phi_{k1k2k3} \left( \frac{\partial D_{k1k2k3i}}{\partial n_i} + \frac{\partial D_{k1k2k3i}}{\partial n_i} \right) + ... + \phi_{12...i-1i+1...N} \left( \frac{\partial D_{12...i-1i+1...N}}{\partial n_i} + \frac{\partial D_{12...i...N}}{\partial n_i} \right) \]

But due to the independence assumption, \( \partial \left( D_f + D_{j/j} \right) / \partial n_i = 0 \), all terms equal zero. Since the same arguments hold for all \( n_k, k = 1, ..., N \), and there is a unique solution, we obtain neutrality also for the \( N \) firms case.

The proof for the case in which only a subset of platforms is controlled by the same owner follows the same arguments as before and is therefore omitted. ■
Proof that viewer surplus and advertiser revenue fall with viewer pricing

With viewer pricing, the equilibrium advertising level is the derivative of \( \omega (D_1 \phi_1 + D_2 \phi_2 + D_{12} \phi_{12}) + p_1 D_1 + p_2 D_2 + (p_1 + p_2) D_{12} \) with respect to \( n_i \). By contrast, without viewer pricing the equilibrium advertising level is the derivative of \( \omega (D_1 \phi_1 + D_2 \phi_2 + D_{12} \phi_{12}) \) with respect to \( n_i \). Since \( p_1, p_2 \geq 0 \), \( \partial D_i / \partial n_i < 0 \), \( \partial D_{12} / \partial n_i < 0 \) and \( \partial D_j / \partial n_i = -\partial D_{12} / \partial n_i \), the derivative of \( p_1 D_1 + p_2 D_2 + (p_1 + p_2) D_{12} \) with respect to \( n_i \) is negative. This implies that the first-order condition with respect to \( n_i \) with viewer pricing is negative at the equilibrium value of \( n_i \) without viewer pricing. As a consequence, the equilibrium advertising level with viewer pricing is below the one without viewer pricing. If viewer demand is lower with viewer pricing than without, then advertising revenue is also lower.

We will now show that viewer demand is smaller with viewer pricing. The profit function with viewer pricing can be rearranged to get

\[
D_1 (\omega \phi_1 + p_1) + D_2 (\omega \phi_2 + p_2) + D_{12} (\omega \phi_{12} + p_1 + p_2).
\]

Therefore, for any demand segment, the monopolist has two potential revenue sources, i.e., it can either use advertising or viewer pricing or both.

Suppose that the monopolist uses both advertising and pricing. Since \( \phi_i(n_i) \) and \( \phi_{12}(n_i, n_j) \) are concave in \( n_i \), the per-viewer revenue from advertising is also concave in \( n_i \). By contrast, the per-viewer revenues from pricing \( p_i \) is linear. Since \( \partial D_i / \partial n_i = \gamma \partial D_i / \partial p_i \) and \( \partial D_{12} / \partial n_i = \gamma \partial D_{12} / \partial p_i \), the first marginal unit of revenue must come from advertising due to the curvature of the demand and revenue functions. If advertising were not used for the first unit of revenue, it will be never be used.

Now if the monopolist increases its advertising further, at some point the marginal revenue from viewer pricing equals the marginal revenue from advertising, since otherwise, the monopolist will not use both revenue sources. At this point, the monopolist will use viewer pricing as well.

Let us now consider the monopolist’s optimal advertising level when pricing is not possible, denoted by \( n_i^* \). If the marginal per-viewer revenue of viewer pricing is lower than that of advertising even at this point, viewer pricing will not be used. Therefore, the optimal solution with and without viewer pricing is the same. Hence, viewer surplus and advertising revenue are unchanged. By contrast, if viewer pricing will be used, we have that at \( n_i^* \) the marginal per-viewer revenue with pricing must be (weakly) larger than without. In addition, we know that the monopolist can induce the same aggregate demand by increasing \( n_i \) by one unit and by increasing \( p_i \) by \( \Delta p_i = \gamma \Delta n_i \). This implies that at the point \( n_i = n_i^* \) and \( p_i = 0 \), the monopolist obtains a larger marginal revenue with viewer pricing. Therefore, the monopolist optimally raises either \( p_i \), inducing less demand than without viewer pricing. As a consequence, viewer surplus and advertising revenue both fall. ■

11.3 Empirical Analysis

Here we empirically investigate the link between entry and correlation in advertising level. As our data is limited, we regard this exercise as providing suggestive evidence, as opposed to a careful empirical analysis of the investigated issues.

The dataset are provided by Kagan-SNL a highly regarded proprietary source for information on broadcasting markets. It consists of an unbalanced panel data set of 68 basic cable channels from 1989 to 2002. The channels cover almost all cable industry advertising revenues (75% of all revenue is generated
by the twenty biggest networks in our data set). We know the date for each new network launch within our sample period (a total of 43 launches), and for each network active in each year we have information on the average number of 30-second advertising slots per hour of programming (in jargon ‘avails’). We also have a good coverage for other network variables, such as subscribers, programming expenses and ratings.

We first use our panel data set to study the relationship between the avails broadcasted by each channel and the number of incumbents. As our model characterizes the effects of varying competition, we consider each channel within its own competitive environment. That is, we define a relevant market segment for each of the 68 channels. The hypothesis is that channels with content tailored to the same segment ‘compete’ for viewers and advertisers. For this purpose, we divide channels in three segments: (i) sports channels (henceforth Sports), (ii) channels broadcasting mainly movies and TV series (henceforth Movies&Series), and (iii) all remaining channels, which is used as a reference group. To test whether viewer preference correlation affects the relationship between entry and advertising levels, we estimate separate parameters for the Sports and the Movies&Series segments. Our working assumption is that the viewers’ preferences within these segments are positively correlated. Our model predicts that avails would fall after entry in the Sports and Movies&Series segments relative to the reference group.32,33

We use two different empirical approaches, that lead us to similar conclusions. First, we use a panel analysis, that pools all channel-year observations from 1989-2002, so it relies on within- and across-channel variation.

We estimate the following linear regression model:

$$\log(\text{Avails}_{it}) = \beta \times \text{Platforms}_{it} + \beta_M \times \text{Platforms}_{it} \times \text{MoviesSeries dummy}$$
$$+ \beta_S \times \text{Platforms}_{it} \times \text{Sports dummy} + \gamma \times x_{it} + \alpha_i + \delta_t + \epsilon_{it},$$

where $\text{Avails}_{it}$ is the average number in year $t$ of 30-second advertising slots per hour of programming by channel $i$, $\text{Platforms}_{it}$ is the number of channels in channel $i$’s segment at the end of year $t$, $\text{Sports dummy}$ and $\text{MoviesSeries dummy}$ are dummy variables equal to 1 when channel $i$ belongs to the Sport and to the Movies&Series segments respectively (and zero otherwise), $x_{it}$ is a vector of channel-time controls, $\alpha_i$ is a channel fixed effect and $\delta_t$ is a time fixed effect. Given that the dependent variable is transformed in logs, while the main explanatory variable is measured in units of channels, $\beta$ has the following interpretation: when a new channel enters the control segment, the incumbents increase their 30-second advertising slots by 100\%$. The coefficients $\beta_M$ and $\beta_S$ measure the additional effect that the number of channels has on the avails in the Movies&Series and Sports segments respectively.

Table 1 reports the estimation results when we restrict the coefficient on the number of channels to be homogeneous across segments. We find evidence that entry is associated with an increase in the

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32This implies that we are testing the predictions of Propositions 5 and 6 and not from the viewer pricing section. This is because viewer prices were not particularly important during our sample period (see for example, Strömberg (2004)). In addition, viewer prices were highly regulated in the 1990s, see Hazlett (1997).

33We note that we intended to create a separate News segment as well, as this segment provides a natural counterpart to the others in that viewer preferences can be reasonably assumed to be negatively correlated. Unfortunately, the number of channels here is too small to obtain statistically meaningful results. The point estimates one can obtain are consistent with Propositions 5 and 6 - contact the authors for details.
advertising levels on incumbent channels. The coefficient is positive and significant across almost all specifications. Starting from the single variable model in column (1), we progressively add controls and fixed effects: column (2) controls for the real GDP to capture the business cycle’s effect on the advertising market, starting from column (3) we report estimates for a fixed-effect model where the units of observations are the single channels. From column (4) we introduce time dummies, while in columns (5) and (6) we add channel-time controls: the channel’s share of revenues in its segment and its rating. Since we only have US data, the real GDP control is dropped whenever time controls are included. All regressions are estimated with robust standard errors. The average effect estimated is on the order of 1%.

Table 2 reports the estimation results when we allow for heterogeneous effects in the number of channels across segments. The coefficients of interest are $\beta_M$ and $\beta_S$. Given our theory and our assumption that preferences are correlated within segments, we expect these coefficients to be negative. That is, we expect the effect of entry within the Sports and Movies&Series segment to be diminished compared to the average industry effect (and possibly negative over all). Indeed, the coefficients have the expected sign in all regressions: the effect of the number of channels on advertising levels is positive for channels in the reference group ($\beta$ is again positive and significant), while it is significantly lower for channels in the other two segments. This additional negative effect is particularly strong for Movies&Series where $|\beta_M| > |\beta|$ in almost all specifications. Standard errors are clustered at the segment level.

To summarize, we obtain evidence of a positive relationship between entry and advertising levels. We also find a systematic reduced impact of entry on advertising levels within the same market segments.
Based on our theory, we speculate that this difference comes from viewers’ tastes for content which induce a good deal of overlap among viewers of the channels belonging to each of these segments. We leave a more careful empirical investigation of these issues to future research.

The regressions above have the advantage of pooling data on different channels without taking a stance on the time it takes for entry to impact the incumbent choices. However, this strategy does not allow to account for within channel omitted variables that vary over time. These variables may also operate at the segment level. To account for this, as an alternative way to address the same issues empirically, we also estimate a model for entry episodes, where our sample is now reduced to the periods when a given segment experiences the entry of a new channel. We estimate the following model:

\[
\Delta \log(\text{Avails}_{it}) = \beta + \beta_M \times \text{MoviesSeries\_dummy} + \beta_S \times \text{Sports\_dummy} \\
+ \gamma \times x_{it} + \delta_t + \epsilon_{it}
\]
This model can be obtained by first differencing the previous model around the years when entry occurs. In fact $\Delta \log(\text{Avails}_{it}) = \log(\text{Avails}_{it+1}) - \log(\text{Avails}_{it-1})$ and the effect of entry (changed number of incumbents) is captured by the constant terms. Channel fixed effects are now excluded (as they cancel out in taking first differences), but we keep time fixed effects and also add some channel controls. The constant $\beta$ measures the effect of entry on the reference group (infotainment), while $\beta_M$ and $\beta_S$ measure the additional effect for the Movies&Series and Sports segments, respectively. The estimates reported in Table 3 confirm our previous results: entry episodes are associated with an increase in the quantity of avails in the reference group, while the effect is lower in the Sports and Movies&Series segments. Since there are half as many observations in this setup, the point estimates are less precisely estimated than in Tables 1 and 2. Furthermore, because here we are looking at the effect one year after entry ($t+1$), the magnitude of the parameters are notably bigger. The point estimate of the % variation in avails due to an additional channel is on the order of 5% in column (3). Notably, the interaction term that captures the differential impact of entry in sports is 11% less than the industry average. The difference is statistically and economically significant.

Table 3: Entry Episodes - Average Effect and Effect by Segment

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<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
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<td>MoviesSeries dummy</td>
<td>-0.0327***</td>
<td>-0.0494**</td>
<td>-0.0314</td>
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<tr>
<td></td>
<td>(0.000919)</td>
<td>(0.00841)</td>
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<tr>
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<tr>
<td></td>
<td>(0.000457)</td>
<td>(0.00460)</td>
<td>(0.0222)</td>
<td></td>
</tr>
<tr>
<td>$\Delta \text{GDP}[t-1,t+1]$</td>
<td>0.00380</td>
<td>0.00328*</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.00133)</td>
<td>(0.00103)</td>
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</tr>
<tr>
<td>Rating</td>
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<tr>
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<td></td>
<td></td>
<td>(0.00128)</td>
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<tr>
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<tr>
<td>Constant</td>
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<td>0.0178</td>
<td>0.0615**</td>
<td>0.263***</td>
</tr>
<tr>
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<td>(0.00901)</td>
<td>(0.00875)</td>
<td>(0.000141)</td>
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<td>YES</td>
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</tr>
</tbody>
</table>

Robust standard errors in parentheses
*** p<0.01, ** p<0.05, * p<0.1
Figure 5: Exclusive GDN consumers in the Auto-Insurance market (2011)